# ME 104: Homework 4 Solutions

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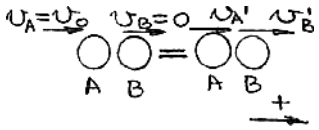
## Chapter 13, Solution 160




(*a*) First collision (between *A* and *B*).

The total momentum is conserved.

$$mv_A + mv_B = mv_A' + mv_B'$$

$$v_0 = v_A' + v_B' \qquad (1)$$

Relative velocities.

$$\left(v_A - v_B\right)e = \left(v_B' - v_A'\right)$$

$$v_0 e = v_B' - v_A' \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

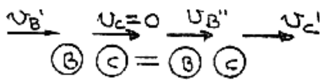
$$v_A' = \frac{v_0(1-e)}{2} \blacktriangleleft$$

$$v_B' = \frac{v_0(1+e)}{2} \blacktriangleleft$$

(*b*) Second collision (between *B* and *C*).

The total momentum is conserved.

$$mv_B' + mv_C = mv_B'' + mv_C'$$




Using the result from (*a*) for $v_B'$

$$\frac{v_0(1+e)}{2} + 0 = v_B'' + v_C' \qquad (3)$$

Relative velocities.

$$(v_B' - 0)e = v_C' - v_B''$$

Substituting again for $v_B'$ from (*a*)

$$v_0 \frac{(1+e)}{2}(e) = v_C' - v_B'' \qquad (4)$$

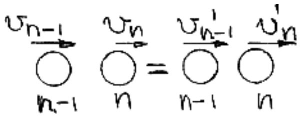
Solving equations (3) and (4) simultaneously,

$$v_C' = \frac{1}{2}\left[\frac{v_0(1+e)}{2} + v_0(1+e)\frac{(e)}{2}\right]$$

$$v_C' = \frac{v_0(1+e)^2}{4} \blacktriangleleft$$

$$v_B'' = \frac{v_0(1-e^2)}{4} \blacktriangleleft$$

## PROBLEM 13.160 (Continued)




($c$) For $n$ spheres

$n$ balls

$n-1^{\text{th}}$ collision,

we note from the answer to Part ($b$) with $n = 3$

$$v'_n = v'_3 = v'_C = \frac{v_0(1+e)^2}{4}$$

or $$v'_3 = \frac{v_0(1+e)^{(3-1)}}{2^{(3-1)}}$$

Thus, for $n$ balls

$$v'_n = \frac{v_0(1+e)^{(n-1)}}{2^{(n-1)}} \blacktriangleleft$$

($d$) For $n = 6$, $e = 0.95$,

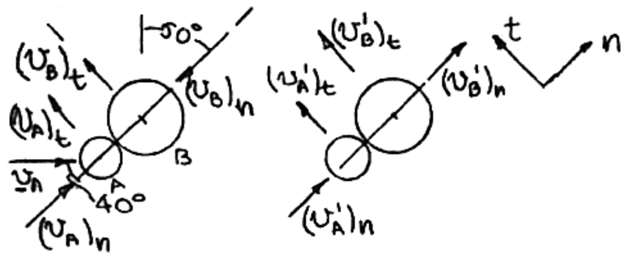
from the answer to Part ($c$) with $n = 6$

$$v'_B = \frac{v_0(1+0.95)^{(6-1)}}{2^{(6-1)}}$$

$$= \frac{v_0(1.95)^5}{(2)^5}$$

$$v'_B = 0.881v_0 \blacktriangleleft$$

## Chapter 13, Solution 165




Before After

$$v_A = 6 \text{ m/s}$$
$$(v_A)_n = (6)(\cos 40°) = 4.596 \text{ m/s}$$
$$(v_A)_t = -6(\sin 40°) = -3.857 \text{ m/s}$$
$$v_B = (v_B)_n = -4 \text{ m/s}$$
$$(v_B)_t = 0$$

<u>*t*-direction</u>.

<u>Total momentum conserved</u>.

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_B)_t + m_B(v'_B)_t$$
$$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)t$$
$$-2.314 \text{ m/s} = 0.6\,(v'_A)_t + (v'_B)_t \qquad (1)$$

<u>Ball *A* alone</u>.

<u>Momentum conserved</u>.

$$m_A\left(v_A\right)_t = m_A\left(v'_A\right)_t \qquad -3.857 = (v'_A)_t$$
$$(v'_A)_t = -3.857 \text{ m/s} \qquad (2)$$

Replacing $(v'_A)_t$ in (2) in Eq. (1)

$$-2.314 = (0.6)(-3.857) + (v'_B)_t$$
$$-2.314 = -2.314 + (v'_B)_t$$
$$(v'_B)_t = 0$$

<u>*n*-direction</u>

<u>Relative velocities</u>.

$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$
$$6.877 = (v'_B)_n - (v'_A)_n \qquad (3)$$

## PROBLEM 13.165 (Continued)

Total momentum conserved.

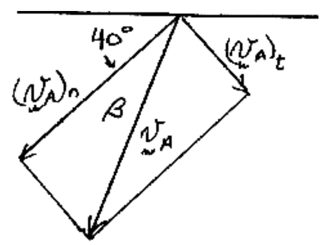
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
$$(0.6\text{ kg})(4.596\text{ m/s}) + (1\text{ kg})(-4\text{ m/s}) = (1\text{ kg})(v'_B)_n + (0.6\text{ kg})(v'_A)_n$$
$$-1.2424 = (v'_B)_n + 0.6\,(v'_A)_n \qquad (4)$$

Solving Eq. (4) and (3) simultaneously,

$$(v'_A)_n = 5.075\text{ m/s}$$
$$(v'_B)_n = 1.802\text{ m/s}$$

Velocity of *A*.

$$\tan\beta = \frac{|(v_A)_t|}{|(v_A)_n|} = \frac{3.857}{5.075}$$
$$\beta = 37.2^\circ \quad \beta + 40^\circ = 77.2^\circ$$
$$v'_A = \sqrt{(3.857)^2 + (5.075)^2} = 6.37\text{ m/s}$$

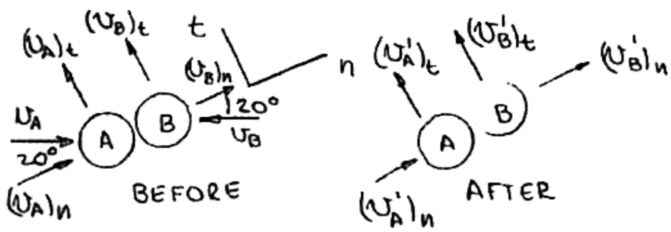



$\mathbf{v}'_A = 6.37\text{ m/s} \measuredangle 77.2^\circ$ ◀

Velocity of *B*.

$\mathbf{v}'_B = 1.802\text{ m/s} \measuredangle 40^\circ$ ◀

## Chapter 13, Solution 166




$$(v_A)_n = (3 \text{ m/s}) \cos 20° = 2.819 \text{ m/s}$$
$$(v_A)_t = (-3 \text{ m/s}) \sin 20° = -1.0261 \text{ m/s}$$

$m_A = m_B$

$$(v_B)_n = (-3 \text{ m/s}) \cos 20° = -2.819 \text{ m/s}$$
$$(v_B)_t = (3 \text{ m/s}) \sin 20° = 1.0261 \text{ m/s}$$

*t*-direction

Momentum of *A* is conserved.

$$m_A(v_A)_t = m_A(v'_A)_t \quad -1.0261 = (v'_A)_t$$
$$(v'_A)_t = -1.0261 \text{ m/s}$$

Momentum of *B* is conserved.

$$m_B(v_B)_t = m_B(v'_B)t \quad 1.0261 = (v'_B)_t$$
$$(v'_B)_t = 1.0261 \text{ m/s}$$

*n*-direction

Total momentum is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$m_A = m_B$

$$2.819 - 2.819 = (v'_A)_n + (v'_B)_n$$
$$(v'_A)_n = -(v'_B)_n$$

## PROBLEM 13.166 (Continued)

Relative velocities (coefficient of restitution).

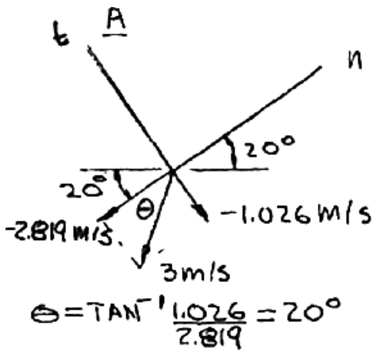
$e = 1$

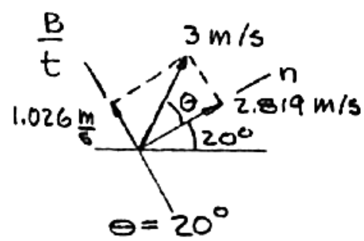
$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[2.819 - (-2.819)](1) = (v'_B)_n - (v'_A)_n$$

$(v'_A)_n = -(v'_B)_n$

$$(v'_B)_n - (v'_A)_n = 5.638$$
$$2(v'_A)_n = -5.638$$

$$(v'_A)_n = -2.819 \text{ m/s} \qquad (v'_B)_n = 2.819 \text{ m/s}$$

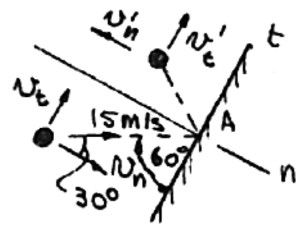



$\mathbf{v}'_A = 3.00$ m/s ↙ 40° ◀

$\mathbf{v}'_B = 3.00$ m/s ↗ 40° ◀

## Chapter 13, Solution 170

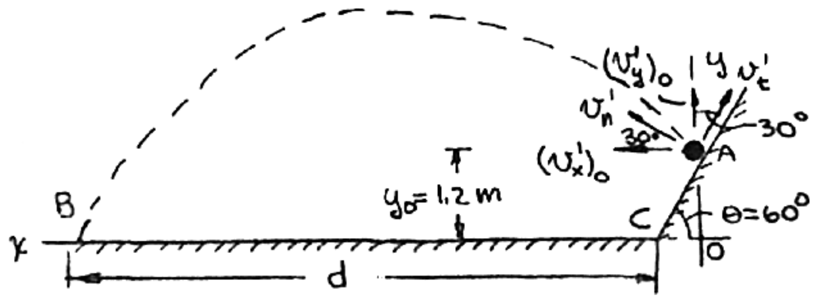



Momentum in *t* direction is conserved.

$$mv\sin 30° = mv_t'$$
$$(15)(\sin 30°) = v't$$
$$v_t' = 7.5 \text{ m/s}$$

Coefficient of restitution in *n*-direction.

$$(v\cos 30°)e = v_n'$$
$$(15)(\cos 30°)(0.9) = v_n'$$
$$v_n' = 11.69 \text{ m/s}$$




*Writing v′ in terms of x and y components*

$$(v_x')_0 = v_n'\cos 30° - v_t'\sin 30°$$
$$(v_x')_0 = (11.69)(\cos 30°) - (7.5)(\sin 30°) = 6.374 \text{ m/s}$$
$$(v_y')_0 = v_n'\sin 30° + v_t'\cos 30°$$
$$(v_y')_0 = (11.69)(\sin 30°) + (7.5)(\cos 30°) = 12.340 \text{ m/s}$$

Motion of a projectile. (origin at 0)

$$y = y_0 + (v_y')_0 t - \frac{(gt^2)}{2}$$
$$y = 1.2 + (12.340 \text{ m/s})t - (9.81 \text{ m/s}^2)\frac{t^2}{2}$$

## PROBLEM 13.170 (Continued)

Time to reach Point $B$ $\quad (y_B = 0)$

$$0 = 1.2 + 12.340 t_B - \left(\frac{9.81}{2}\right) t_B^2 \qquad t_B = 2.610 \text{ s}$$

$$x = x_0 + (v_x')_0 t$$

$$x = 0 + 6.374t$$

$$x_B = (6.374)(t_B)$$

$$= (6.374 \text{ m/s})(2.610 \text{ s})$$

$$x_B = 16.63 \text{ m}$$

$$d = x_B - 1.2 \cot 60°$$

$$= 15.94 \text{ m}$$

$d = 15.94$ m ◀

## Chapter 12, Solution 102

Using Eq. (12.39), $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and $$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But $$\theta_B = \theta_A + 180°,$$

so that $$\cos\theta_A = -\cos\theta_B.$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$$

## Chapter 12, Solution 103

For earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

$$r_A = 3960 + 40.3 = 4000.3 \text{ mi} = 21.1216 \times 10^6 \text{ ft}$$

$$r_B = 3960 + 336 = 4296 \text{ mi} = 22.6829 \text{ ft}$$

Elliptic trajectory.

Using Eq. (12.39), $\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$ and $\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_A = -\cos\theta_B$

Adding,

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2GM}{h^2}$$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}}$$

$$= \sqrt{\frac{(2)(14.077 \times 10^{15})(21.1216 \times 10^6)(22.6829 \times 10^6)}{43.8045 \times 10^6}}$$

$$= 554.911 \times 10^9 \text{ ft}^2/\text{s}$$

(*a*) Speed $v_0$ at *A*.

$$v_0 = v_A = \frac{h}{r_A} = \frac{554.911 \times 10^9}{21.1216 \times 10^6}$$

$v_0 = 26.3 \times 10^3$ ft/s ◀

$$(v_B)_1 = \frac{h}{r_A}$$

$$= \frac{554.911 \times 10^9}{22.6829 \times 10^6}$$

$$= 24.464 \times 10^3 \text{ ft/s}$$

For a circular orbit through Point *B*,

$$(v_B)_{\text{circ}} = \sqrt{\frac{GM}{r_B}}$$

$$= \sqrt{\frac{14.077 \times 10^{15}}{22.6829 \times 10^6}}$$

$$= 24.912 \times 10^3 \text{ ft/s}$$

(*b*) Increase in speed at Point *B*.

$$\Delta v_B = (v_B)_{\text{circ}} - (v_B)_1$$

$$= 448 \text{ ft/s}$$

$\Delta v_B = 448$ ft/s ◀

## Chapter 12, Solution 105

First note
$$R_{\text{earth}} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$$
$$r_A = 202 \times 10^6 \text{ mi} = 1066.56 \times 10^9 \text{ ft}$$
$$r_B = 92 \times 10^6 \text{ mi} = 485.76 \times 10^9 \text{ ft}$$

From the solution to Problem 12.102, we have for any elliptic orbit about the sun

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{\text{sun}}}{h^2}$$

(*a*) For the elliptic orbit *AB*, we have

$$r_1 = r_A, \quad r_2 = r_B, \quad h = h_A = r_A v_A$$

Also,
$$GM_{\text{sun}} = G[(332.8 \times 10^3) M_{\text{earth}}]$$
$$= gR_{\text{earth}}^2 (332.8 \times 10^3) \qquad \text{using Eq. (12.30).}$$

Then
$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR_{\text{earth}}^2 (332.8 \times 10^3)}{(r_A v_A)^2}$$

or
$$v_A = \frac{R_{\text{earth}}}{r_A} \left( \frac{665.6g \times 10^3}{\frac{1}{r_A} + \frac{1}{r_B}} \right)^{1/2}$$
$$= \frac{3960 \text{ mi}}{202 \times 10^6 \text{ mi}} \left( \frac{665.6 \times 10^3 \times 32.2 \text{ ft/s}^2}{\frac{1}{1066.56 \times 10^9 \text{ ft}} + \frac{1}{485.76 \times 10^9 \text{ ft}}} \right)^{1/2}$$
$$= 52,431 \text{ ft/s}$$

or
$$v_A = 52.4 \times 10^3 \text{ ft/s} \blacktriangleleft$$

(*b*) From Part (*a*), we have

$$2GM_{\text{sun}} = (r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)$$

Then, for any other elliptic orbit about the sun, we have

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{(r_A v_A)^2 \left( \frac{1}{r_A} + \frac{1}{r_B} \right)}{h^2}$$

For the elliptic transfer orbit $AB'$, we have

$$r_1 = r_A, \quad r_2 = r_{B'}, \quad h = h_{\text{tr}} = r_A (v_A)_{\text{tr}}$$

Then $$\frac{1}{r_A}+\frac{1}{r_{B'}}=\frac{(r_Av_A)^2\left(\frac{1}{r_A}+\frac{1}{r_B}\right)}{[r_A(v_A)_{\text{tr}}]^2}$$

or $$(v_A)_{\text{tr}}=v_A\left(\frac{\frac{1}{r_A}+\frac{1}{r_B}}{\frac{1}{r_A}+\frac{1}{r_{B'}}}\right)^{1/2}=v_A\left(\frac{1+\frac{r_A}{r_B}}{1+\frac{r_A}{r_{B'}}}\right)^{1/2}$$

$$=(52,431\text{ ft/s})\left(\frac{1+\frac{202}{92}}{1+\frac{202}{85.5}}\right)^{1/2}$$

$$=51{,}113\text{ ft/s}$$

Now $$h_{\text{tr}}=(h_A)_{\text{tr}}=(h_{B'})_{\text{tr}}:\quad r_A(v_A)_{\text{tr}}=r_{B'}(v_{B'})_{\text{tr}}$$

Then $$(v_{B'})_{\text{tr}}=\frac{202\times10^6\text{ mi}}{85.5\times10^6\text{ mi}}\times51{,}113\text{ ft/s}=120{,}758\text{ ft/s}$$

For the elliptic orbit $A'B'$, we have

$$r_1=r_{A'},\quad r_2=r_{B'},\quad h=r_{B'}v_{B'}$$

Then $$\frac{1}{r_{A'}}+\frac{1}{r_{B'}}=\frac{(r_Av_A)^2\left(\frac{1}{r_A}+\frac{1}{r_B}\right)}{(r_{B'}v_{B'})^2}$$

or $$v_{B'}=v_A\frac{r_A}{r_{B'}}\left(\frac{\frac{1}{r_A}+\frac{1}{r_B}}{\frac{1}{r_{A'}}+\frac{1}{r_{B'}}}\right)^{1/2}$$

$$=(52,431\text{ ft/s})\frac{202\times10^6\text{ mi}}{85.5\times10^6\text{ mi}}\left(\frac{\frac{1}{202\times10^6}+\frac{1}{92\times10^6}}{\frac{1}{164.5\times10^6}+\frac{1}{85.5\times10^6}}\right)^{1/2}$$

$$=116,862\text{ ft/s}$$

Finally, $$(v_A)_{\text{tr}}=v_A+\Delta v_A$$

or $$\Delta v_A=(51{,}113-52,431)\text{ ft/s}$$

or $$|\Delta v_A|=1318\text{ ft/s}\ \blacktriangleleft$$

and $$v_{B'}=(v_{B'})_{\text{tr}}+\Delta v_B$$

or $$\Delta v_{B'}=(116,862-120,758)\text{ ft/s}$$
$$=-3896\text{ ft/s}$$

or $$|\Delta v_B|=3900\text{ ft/s}\ \blacktriangleleft$$

## Chapter 12, Solution 117

First we note
$$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$$
$$r_A = (3960 + 350) \text{ mi} = 4310 \text{ mi}$$
$$= 22.7568 \times 10^6 \text{ ft}$$
$$r_B = (3960 + 75) \text{ mi} = 4035 \text{ mi}$$

For the circular orbit, we have
$$v_{\text{circ}} = \sqrt{\frac{gR^2}{r_A}} \qquad \text{[Eq. (12.44)]}$$
$$= 20.9088 \times 10^6 \text{ ft} \left( \frac{32.2 \text{ ft/s}^2}{22.7568 \times 10^6 \text{ ft}} \right)^{1/2}$$
$$= 24{,}871 \text{ ft/s}$$

Now
$$(v_A)_{AB} = v_{\text{circ}} + \Delta v_A = (24{,}871 - 500) \text{ ft/s}$$
$$= 24{,}371 \text{ ft/s}$$

For the elliptic descent trajectory, we have
$$\frac{1}{r} = \frac{GM}{h^2} + C \cos\theta \qquad \text{[Eq. (12.39)]}$$

Noting that Point $A$ is at the apogee of this trajectory, we have

at $A$, $\theta = 180°$:
$$\frac{1}{r_A} = \frac{GM}{h^2} - C$$

or
$$C = \frac{GM}{h^2} - \frac{1}{r_A}$$

at $B$, $\theta = \theta_B = 180° - \measuredangle AOB$:
$$\frac{1}{r_B} = \frac{GM}{h^2} + C \cos \theta_B$$

or
$$C = \frac{1}{\cos \theta_B} \left( \frac{1}{r_B} - \frac{GM}{h^2} \right)$$

Then
$$\frac{GM}{h^2} - \frac{1}{r_A} = \frac{1}{\cos \theta_B} \left( \frac{1}{r_B} - \frac{GM}{h^2} \right)$$

or
$$\cos \theta_B = \frac{\frac{1}{r_B} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{r_A}}$$

Now
$$h = (h_A)_{AB} = r_A (v_A)_{AB}$$

and
$$GM = gR^2 \qquad \text{[Eq. (12.30)]}$$

## PROBLEM 12.117 (Continued)

From above, $$gR^2 = r_A(v_{\text{circ}})^2 \qquad \text{[Eq. (12.44)]}$$

Then $$\frac{GM}{h^2} = \frac{r_A(v_{\text{circ}})^2}{[r_A(v_A)_{AB}]^2} = \frac{1}{r_A}\left[\frac{v_{\text{circ}}}{(v_A)_{AB}}\right]^2$$

so that $$\cos\theta_B = \frac{\frac{1}{r_B} - \frac{1}{r_A}\left[\frac{v_{\text{circ}}}{(v_A)_{AB}}\right]^2}{\frac{1}{r_A}\left[\frac{v_{\text{circ}}}{(v_A)_{AB}}\right]^2 - \frac{1}{r_A}} = \frac{\frac{r_A}{r_B} - \left[\frac{v_{\text{circ}}}{(v_A)_{AB}}\right]^2}{\left[\frac{v_{\text{circ}}}{(v_A)_{AB}}\right]^2 - 1}$$

$$= \frac{\frac{4310\text{ mi}}{4035\text{ mi}} - \left(\frac{24{,}871\text{ ft/s}}{43{,}371\text{ ft/s}}\right)^2}{\left(\frac{24{,}871\text{ ft/s}}{24{,}371\text{ ft/s}}\right)^2 - 1}$$

$$= 0.64411$$

or $$\theta_B = 49.901°$$

Finally, $$\measuredangle AOB = 180° - 49.901°$$

or $$\measuredangle AOB = 130.1° \blacktriangleleft$$

## Chapter 12, Solution 133

For Earth, $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM = gR^2 = (32.2)(20.909 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

(*a*) For the elliptic orbit, $r_A = 3960 + 40 = 4000 \text{ mi} = 21.12 \times 10^6 \text{ ft}$

$$r_B = 3960 + 170 = 4130 \text{ mi} = 21.8064 \times 10^6 \text{ ft}$$

$$a = \frac{1}{2}(r_A + r_B) = 21.5032 \times 10^6 \text{ ft}$$

$$b = \sqrt{r_A r_B} = 21.4605 \times 10^6 \text{ ft}$$

Using Eq. 12.39, $\dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A$

and $\dfrac{1}{r_B} = \dfrac{GM}{r_B^2} + C\cos\theta_B$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2a}{b^2} = \dfrac{2GM}{h^2}$

or $h = \sqrt{\dfrac{GMb^2}{a}}$

<u>Periodic time</u>. $\tau = \dfrac{2\pi ab}{h} = \dfrac{2\pi ab\sqrt{a}}{\sqrt{GMb^2}} = \dfrac{2\pi a^{3/2}}{\sqrt{GM}}$

$$\tau = \frac{2\pi(21.5032 \times 10^6)^{3/2}}{\sqrt{14.077 \times 10^{15}}} = 5280.6 \text{ s} = 1.4668 \text{ h}$$

The time to travel from *A* to *B* is one half the periodic time

$\tau_{AB} = 0.7334 \text{ h}$ $\qquad \tau_{AB} = 44.0 \text{ min}$ ◀

(*b*) For the circular orbit, $a = b = r_B = 21.4065 \times 10^6 \text{ ft}$

$$\tau_{\text{circ}} = \frac{2\pi a^{3/2}}{\sqrt{GM}} = \frac{2\pi(21.8064 \times 10^6)^{3/2}}{\sqrt{14.077 \times 10^{15}}} = 5393 \text{ s}$$

$\tau_{\text{circ}} = 1.498 \text{ h}$ $\qquad \tau_{\text{circ}} = 89.9 \text{ min}$ ◀