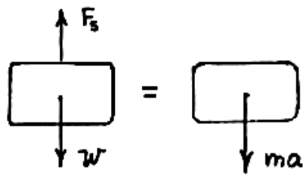
ME 104: Homework 3 Solutions

Andrew Gray: University of California, Berkeley

Chapter 12, Solution 4

Assume $g = 32.2 \text{ ft/s}^2$

(*a*)



$$m = \frac{W}{g}$$

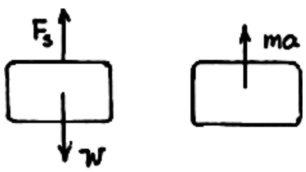
$$+\downarrow \Sigma F = ma: \quad W - F_s = \frac{W}{g}a$$

$$W\left(1 - \frac{a}{g}\right) = F_s$$

or $$W = \frac{F_s}{1 - \frac{a}{g}} = \frac{14.1}{1 - \frac{4}{32.2}}$$

$W = 16.10 \text{ lb}$ ◀

(*b*)



$$m = \frac{W}{g} = \frac{7.4635}{32.2} = 0.232 \text{ lb}\cdot\text{s}^2/\text{ft}$$

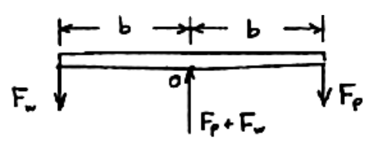
$$\Sigma F = ma: \quad F_s - W = \frac{W}{g}a$$

$$F_s = W\left(1 + \frac{a}{g}\right)$$

$$= 16.10\left(1 + \frac{4}{32.2}\right)$$

$F_s = 18.10 \text{ lb}$ ◀

For the balance system *B*,



$$\Sigma M_0 = 0: \quad bF_w - bF_p = 0$$

$$F_w = F_p$$

But $$F_w = W_w\left(1 + \frac{a}{g}\right)$$

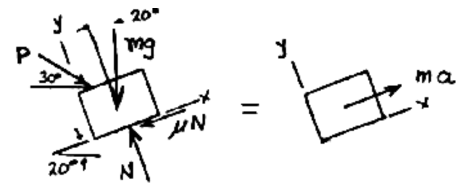
and $$F_p = W_p\left(1 + \frac{a}{g}\right)$$

so that $$W_w = W_p$$

and $$m_w = \frac{W_p}{g} = \frac{16.10}{32.2}$$

$m_w = 0.500 \text{ lb}\cdot\text{s}^2/\text{ft}$ ◀

Chapter 12, Solution 9



Kinematics: Uniformly accelerated motion. $(x_0 = 0,\ v_0 = 0)$

$$x = x_0 + v_0 t + \frac{1}{2}at^2,$$

or $$a = \frac{2x}{t^2} = \frac{(2)(5)}{(10)^2} = 0.100 \text{ m/s}^2$$

$$+\nwarrow \Sigma F_y = 0: \quad N - P \sin 50° - mg \cos 20° = 0$$

$$N = P \sin 50° + mg \cos 20°$$

$$+\nearrow \Sigma F_x = ma: \quad P \cos 50° - mg \sin 20° - \mu N = ma$$

or $$P \cos 50° - mg \sin 20° - \mu(P \sin 50° + mg \cos 20°) = ma$$

$$P = \frac{ma + mg(\sin 20° + \mu \cos 20°)}{\cos 50° - \mu \sin 50°}$$

For motion impending, set $a = 0$ and $\mu = \mu_s = 0.4$

$$P = \frac{(20)(0) + (20)(9.81)(\sin 20° + 0.4 \cos 20°)}{\cos 50° - 0.4 \sin 50°}$$

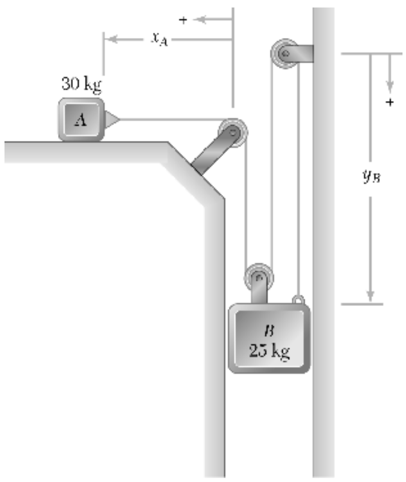
$$= 419 \text{ N}$$ ◀

For motion with $a = 0.100 \text{ m/s}^2$, use $\mu = \mu_k = 0.3$.

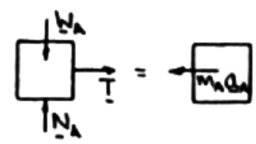
$$P = \frac{(20)(0.100) + (20)(9.81)(\sin 20° + 0.3 \cos 20°)}{\cos 50° - 0.3 \sin 50°}$$

$P = 301$ N ◀

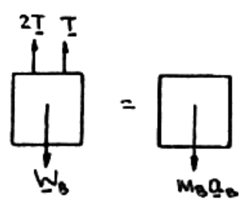
Chapter 12, Solution 11



A:



B:



From the diagram

$$x_A + 3y_B = \text{constant}$$

Then $\qquad v_A + 3v_B = 0$

and $\qquad a_A + 3a_B = 0$

or $\qquad a_A = -3a_B \qquad (1)$

(a) $\quad A$: $\quad \overset{+}{\leftarrow} \Sigma F_x = m_A a_A$: $\qquad -T = m_A a_B$

Using Eq. (1) $\qquad T = 3m_A a_B$

B: $\quad +\!\downarrow \Sigma F_y = m_B a_B$: $\quad W_B - 3T = m_B a_B$

Substituting for T

$$m_B g - 3(3m_A a_B) = m_B a_B$$

or

$$a_B = \frac{g}{1 + 9\dfrac{m_A}{m_B}} = \frac{9.81\ \text{m/s}^2}{1 + 9\dfrac{30\ \text{kg}}{25\ \text{kg}}} = 0.83136\ \text{m/s}^2$$

Then $\qquad \mathbf{a}_A = 2.49\ \text{m/s}^2 \longrightarrow$ ◀

and $\qquad \mathbf{a}_B = 0.831\ \text{m/s}^2 \downarrow$ ◀

(b) $\quad$ We have $\qquad T = 3 \times 30\ \text{kg} \times 0.83136\ \text{m/s}^2$

or $\qquad T = 74.8\ \text{N}$ ◀

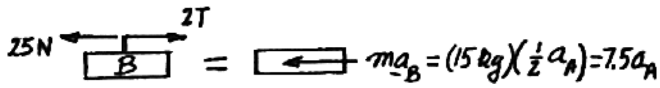
Chapter 12, Solution 18

Referring to solution of Problem 11.51 we note that

$$a_B = -\frac{1}{2}a_A \qquad (1)$$

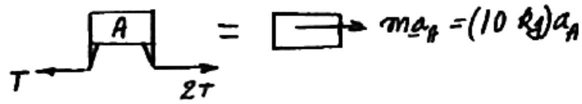
where minus sign indicates that $\mathbf{a}_A$ and $\mathbf{a}_B$ have opposite sense.

Block *B*.



$$\overset{+}{\leftarrow} \Sigma F = ma: \quad 25 - 2T = 7.5a_A \qquad (2)$$

Block *A*.



$$\overset{+}{\rightarrow} \Sigma F = ma: \quad 2T - T = 10a_A$$

$$T = 10a_A \qquad (3)$$

(*a*) Substituting for *T* from (3) into (2):

$$25 - 2(10\,a_A) = 7.5a_A$$

$$25 = 27.5a_A \qquad \mathbf{a}_A = 0.909 \text{ m/s}^2 \rightarrow$$

$$v_A = (v_A)_0 + a_A t = 0 + 0.909(1.2), \qquad \mathbf{v}_A = 1.091 \text{ m/s} \rightarrow \blacktriangleleft$$

(*b*) Substituting $a = 0.909$ into Eq. (1): $\qquad \mathbf{a}_B = 0.455 \text{ m/s}^2 \leftarrow$

$$v_B = (v_B)_0 + a_B t = 0 + 0.455(1.2) \qquad \mathbf{v}_B = 0.545 \text{ m/s} \leftarrow \blacktriangleleft$$

Chapter 12, Solution 23

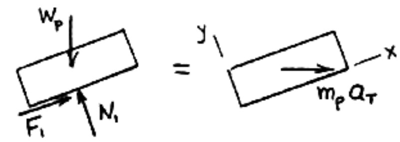
Let $\mathbf{a}_P$ be the acceleration of the plywood, $\mathbf{a}_T$ be the acceleration of the truck, and $\mathbf{a}_{P/T}$ be the acceleration of the plywood relative to the truck.

(*a*) Find the value of $\mathbf{a}_T$ so that the relative motion of the plywood with respect to the truck is impending.

$$a_P = a_T \text{ and } F_1 = \mu_s N_1 = 0.40 N_1$$

$$+\nwarrow \Sigma F_y = m_P a_y: \quad N_1 - W_P \cos 20° = -m_P a_T \sin 20°$$

$$N_1 = m_P (g \cos 20° - a_T \sin 20°)$$



$$+\nearrow \Sigma F_x = m a_x: \quad F_1 - W_P \sin 20° = m_P a_T \cos 20°$$

$$F_1 = m_P (g \sin 20° + a_T \cos 20°)$$

$$m_P (g \sin 20° + a_T \cos 20°) = 0.40\, m_P (g \cos 20° - a_T \sin 20°)$$

$$a_T = \frac{(0.40 \cos 20° - \sin 20°)}{\cos 20° + 0.40 \sin 20°} g$$

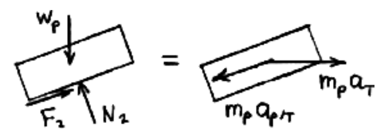
$$= (0.03145)(9.81)$$

$$= 0.309$$

$\mathbf{a}_T = 0.309 \text{ m/s}^2 \longrightarrow$ ◀

(*b*) $$x_{P/T} = (x_{P/T})_o + (v_{P/T})t + \frac{1}{2} a_{P/T} t^2 = 0 + 0 + \frac{1}{2} a_{P/T} t^2$$

$$a_{P/T} = \frac{2 x_{P/T}}{t^2} = \frac{(2)(2)}{(0.9)^2} = 4.94 \text{ m/s}^2$$



$$\mathbf{a}_{P/T} = 4.94 \text{ m/s}^2 \swarrow 20°$$

$$\mathbf{a}_P = \mathbf{a}_T + \mathbf{a}_{P/T} = (a_T \rightarrow) + (4.94 \text{ m/s}^2 \swarrow 20°)$$

$$+\nwarrow F_y = m_P a_y: \quad N_2 - W_P \cos 20° = -m_P a_T \sin 20°$$

$$N_2 = m_P (g \cos 20° - a_T \sin 20°)$$

$$+\nearrow \Sigma F_x = \Sigma m a_x: \quad F_2 - W_P \sin 20° = m_P a_T \cos 20° - m_P a_{P/T}$$

$$F_2 = m_P (g \sin 20° + a_T \cos 20° - a_{P/T})$$

For sliding with friction $\quad F_2 = \mu_k N_2 = 0.30 N_2$

$$m_P (g \sin 20° + a_T \cos 20° - a_{P/T}) = 0.30 m_P (g \cos 20° + a_T \sin 20°)$$

$$a_T = \frac{(0.30 \cos 20° - \sin 20°) g + a_{P/T}}{\cos 20° + 0.30 \sin 20°}$$

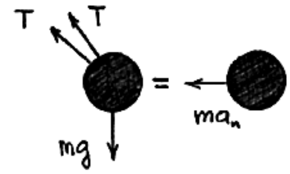
$$= (-0.05767)(9.81) + (0.9594)(4.94)$$

$$= 4.17$$

$\mathbf{a}_T = 4.17 \text{ m/s}^2 \longrightarrow$ ◀

Chapter 12, Solution 39

(*a*) $+\uparrow \Sigma F_y = 0: \quad T\cos\theta_1 + T\cos\theta_2 - W = 0$



$$\cos\theta_2 = \frac{W}{T} - \cos\theta_1$$

$$= \frac{12}{7.6} - \cos 50° = 0.93616$$

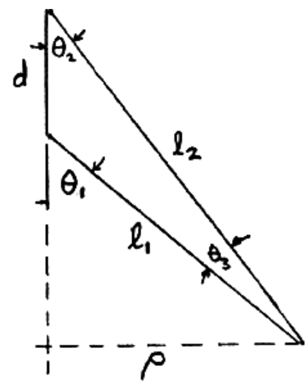
$$\theta_2 = 20.584°$$

$\theta_2 = 20.6°$ ◀

(*b*) $\theta_3 = \theta_1 - \theta_2 = 50° - 20.584° = 29.416°$

$$\frac{\ell_1}{\sin\theta_2} = \frac{d}{\sin\theta_3} \qquad \text{or} \qquad \ell_1 = \frac{d\sin\theta_2}{\sin\theta_3}$$

Radius of horizontal circle



$$\rho = \ell_1 \sin\theta_1 = \frac{d\sin\theta_2\sin\theta_1}{\sin\theta_3}$$

$$= \frac{(30)(\sin 20.584°)(\sin 50°)}{\sin 29.416°}$$

$$= 16.45 \text{ in.} = 1.3709 \text{ ft}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_n: \quad T\sin\theta_1 + T\sin\theta_2 = \frac{mv^2}{\rho}$$

$$v^2 = \frac{\rho T\left(\sin\theta_1 + \sin\theta_2\right)}{m}$$

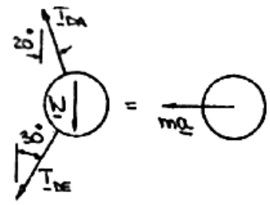
$$= \frac{(1.3709)(7.6)(\sin 50° + \sin 20.584°)}{\frac{12}{32.2}}$$

$$= 31.246 \text{ ft}^2/\text{s}^2$$

$v = 5.59$ ft/s ◀

Chapter 12, Solution 43*

SOLUTION



First note $$a = a_n = \frac{v^2}{\rho}$$

where $$\rho = 0.5 \text{ ft}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma: \quad T_{DA} \sin 20° + T_{DE} \sin 30° = \frac{W}{g}\frac{v^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{DA} \cos 20° - T_{DE} \cos 30° - W = 0 \qquad (2)$$

Note that Eq. (2) implies that

(a) when $T_{DE} = (T_{DE})_{\max}, \quad T_{DA} = (T_{DA})_{\max}$

(b) when $T_{DE} = (T_{DE})_{\min}, \quad T_{DA} = (T_{DA})_{\min}$

<u>Case 1: T_{DA} is maximum.</u>

Let $$T_{DA} = 17 \text{ lb}$$

Eq. (2) $$(17 \text{ lb})\cos 20° - T_{DE} \cos 30° - (1.2 \text{ lb}) = 0$$

or $$T_{DE} = 17.06 \text{ lb} \quad \text{unacceptable (. 17 lb)}$$

Now let $$T_{DE} = 17 \text{ lb}$$

Eq. (2) $$T_{DA} \cos 20° - (17 \text{ lb})\cos 30° - (1.2 \text{ lb}) = 0$$

or $$T_{DA} = 16.9443 \text{ lb} \quad \text{O.K.} \quad (\,,\ 17 \text{ lb})$$

$$(T_{DA})_{\max} = 16.9443 \text{ lb}$$
$$(T_{DE})_{\max} = 17 \text{ lb}$$

Eq. (1) $$(v^2)_{(T_{DA})_{\max}} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(16.9443 \sin 20° + 17 \sin 30°) \text{ lb}$$

or $$v_{(T_{DA})_{\max}} = 13.85 \text{ ft/s}$$

Now form $$(\cos 30°)(1) + (\sin 30°)(2)$$

$$T_{DA} \sin 20° \cos 30° + T_{DA} \cos 20° \sin 30° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30°$$

or $$T_{DA} \sin 50° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30° \qquad (3)$$

$v_{\max}$ occurs when $T_{DA} = (T_{DA})_{\max}$

$v_{\max} = 13.85$ ft/s

PROBLEM 12.43* (Continued)

Case 2: T_{DA} is minimum.

Because $(T_{DA})_{\min}$ occurs when $T_{DE} = (T_{DE})_{\min}$,

let $T_{DE} = 0$.

Eq. (2) $$T_{DA} \cos 20° - (1.2 \text{ lb}) = 0$$

or $$T_{DA} = 1.27701 \text{ lb}, \quad 17 \text{ lb} \qquad \text{O.K.}$$

Note: Eq. (3) implies that when $T_{DA} = (T_{DA})_{\min}$, $v = v_{\min}$. Then

Eq. (1) $$(v^2)_{\min} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(1.27701 \text{ lb}) \sin 20°$$

or $$v_{\min} = 2.42 \text{ ft/s}$$

$0 \# \; T_{AB}, T_{BC}, T_{AD}, T_{DE} \# \; 17 \text{ lb}$

when $2.42 \text{ ft/s} \# \; v \# \; 13.85 \text{ ft/s}$ ◀