ME 104: Homework 2 Solutions

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Chapter 11, Solution 34

(*a*) Initial velocity.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v_0 = \frac{x - x_0}{t} - \frac{1}{2}at$$

$$= \frac{220}{10} - \frac{1}{2}(-0.6)(10)$$

$v_0 = 25.9$ m/s ◀

(*b*) Final velocity.

$$v = v_0 + at$$

$$v = 25.0 + (-0.6)(10)$$

$v_f = 19.00$ m/s ◀

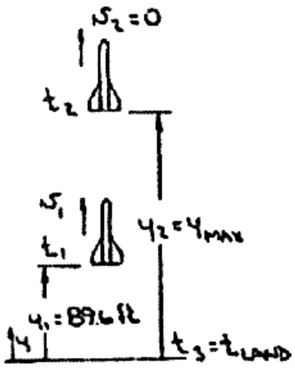
(*c*) Distance traveled during first 1.5 s.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= 0 + (25.0)(1.5) + \frac{1}{2}(-0.6)(1.5)^2$$

$x = 36.8$ m ◀

Chapter 11, Solution 36



(*a*) We have $y = y_1 + v_1 t + \frac{1}{2}at^2$

At t_{land}, $y = 0$

Then $0 = 89.6 \text{ ft} + v_1(16 \text{ s}) + \frac{1}{2}(-32.2 \text{ ft/s}^2)(16 \text{ s})^2$

or $v_1 = 2.52 \text{ ft/s}$ ◀

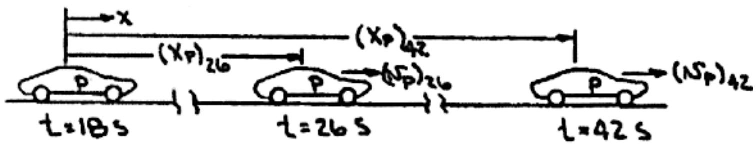
(*b*) We have $v^2 = v_1^2 + 2a(y - y_1)$

At $y = y_{\max}, \quad v = 0$

Then $0 = (252 \text{ ft/s})^2 + 2(-32.2 \text{ ft/s}^2)(y_{\max} - 89.6) \text{ ft}$

or $y_{\max} = 1076 \text{ ft}$ ◀

Chapter 11, Solution 39



$(v_P)_{18} = 0 \qquad (v_P)_{26} = 90 \text{ km/h} = 25 \text{ m/s} \qquad (v_P)_{42} = 90 \text{ km/h} = 25 \text{ m/s}$

(*a*) Patrol car:

For 18 s , t# 26 s: $\quad v_P = 0 + a_P(t - 18)$

At $t = 26$ s: $\quad 25 \text{ m/s} = a_P(26 - 18) \text{ s}$

or $\quad a_P = 3.125 \text{ m/s}^2$

Also, $\quad x_P = 0 + 0(t - 18) - \frac{1}{2}a_P(t - 18)^2$

At $t = 26$ s: $\quad (x_P)_{26} = \frac{1}{2}(3.125 \text{ m/s}^2)(26 - 18)^2 = 100 \text{ m}$

For 26 s , t# 42 s: $\quad x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$

At $t = 42$ s: $\quad (x_P)_{42} = 100 \text{ m} + (25 \text{ m/s})(42 - 26) \text{s}$
$= 500 \text{ m}$

$(x_P)_{42} = 0.5 \text{ km}$ ◀

(*b*) For the motorist's car: $\quad x_M = 0 + v_M t$

At $t = 42$ s, $x_M = x_P$: $\quad 500 \text{ m} = v_M(42 \text{ s})$

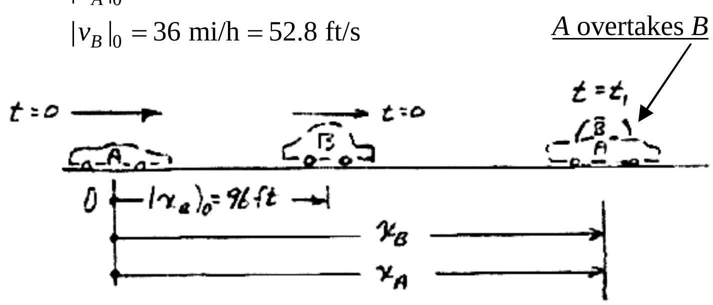
or $\quad v_M = 11.9048 \text{ m/s}$

or $\quad v_M = 42.9 \text{ km/h}$ ◀

Chapter 11, Solution 41

$$a_A = +1.8 \text{ ft/s}^2 \quad a_B = -1.2 \text{ ft/s}^2$$

$$|v_A|_0 = 24 \text{ mi/h} = 35.2 \text{ ft/s}$$
$$|v_B|_0 = 36 \text{ mi/h} = 52.8 \text{ ft/s}$$



Motion of auto A:

$$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t \quad (1)$$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \quad (2)$$

Motion of auto B:

$$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t \quad (3)$$

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \quad (4)$$

(a) A overtakes B at $t = t_1$.

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22 \text{ s} \quad \text{and} \quad t_1 = 15.0546$$

$t_1 = 15.05$ s ◀

Eq. (2): $x_A = 35.2(15.05) + 0.9(15.05)^2$

$x_A = .734$ ft ◀

(b) Velocities when $t_1 = 15.05$ s

Eq. (1): $v_A = 35.2 + 1.8(15.05)$

$v_A = 62.29$ ft/s

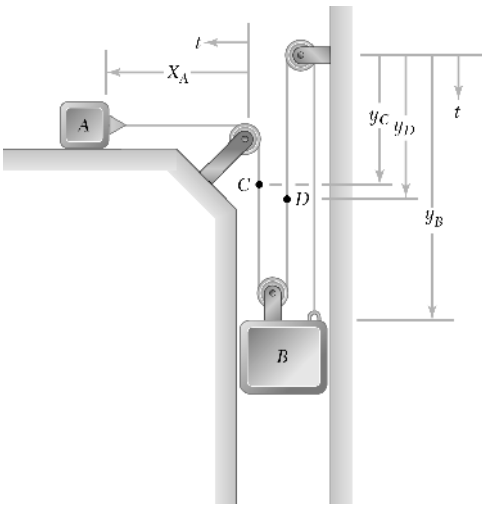
$v_A = 42.5 \text{ mi/h} \rightarrow$ ◀

Eq. (3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74$ ft/s

$v_B = 23.7 \text{ mi/h} \rightarrow$ ◀

Chapter 11, Solution 47



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $$v_A + 3v_B = 0 \quad (1)$$

and $$a_A + 3a_B = 0 \quad (2)$$

(*a*) Substituting into Eq. (1) $6 \text{ m/s} + 3v_B = 0$

or $$\mathbf{v}_B = 2 \text{ m/s} \uparrow \blacktriangleleft$$

(*b*) From the diagram $y_B + y_D = \text{constant}$

Then $v_B + v_D = 0$

$$\mathbf{v}_D = 2 \text{ m/s} \downarrow \blacktriangleleft$$

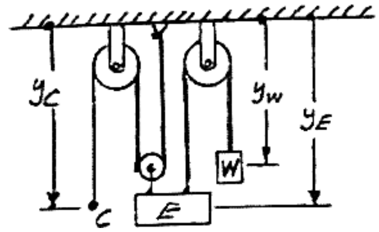
(*c*) From the diagram $x_A + y_C = \text{constant}$

Then $v_A + v_C = 0 \qquad v_C = -6 \text{ m/s}$

Now $v_{C/D} = v_C - v_D = (-6 \text{ m/s}) - (2 \text{ m/s}) = -8 \text{ m/s}$

$$\mathbf{v}_{C/D} = 8 \text{ m/s} \uparrow \blacktriangleleft$$

Chapter 11, Solution 49



Choose the positive direction downward.

(*a*) *Velocity of cable C.*

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $v_E = 15 \text{ ft/s}$

or $v_C = -2v_E = -30 \text{ ft/s}$ $\mathbf{v}_C = 30.0 \text{ ft/s} \uparrow$ ◀

(*b*) *Velocity of counterweight W.*

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -15 \text{ ft/s}$$ $\mathbf{v}_W = 15.00 \text{ ft/s} \uparrow$ ◀

(*c*) *Relative velocity of C with respect to E.*

$$v_{C/E} = v_C - v_E = (-30 \text{ ft/s}) - (+15 \text{ ft/s}) = -45 \text{ ft/s}$$

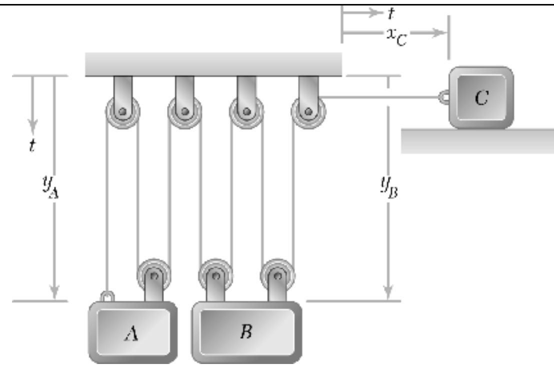
$\mathbf{v}_{C/E} = 45.0 \text{ ft/s} \uparrow$ ◀

(*d*) *Relative velocity of W with respect to E.*

$$v_{W/E} = v_W - v_E = (-15 \text{ ft/s}) - (15 \text{ ft/s}) = -30 \text{ ft/s}$$

$\mathbf{v}_{W/E} = 30.0 \text{ ft/s} \uparrow$ ◀

Chapter 11, Solution 55



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \quad (1)$$

and $$3a_A + 4a_B + a_C = 0 \quad (2)$$

<u>Given</u>: $$\mathbf{v}_B = 20 \text{ mm/s} \downarrow;$$

$$(\mathbf{v}_A)_0 = 30 \text{ mm/s} \uparrow$$

(*a*) Substituting into Eq. (1) at $t = 0$

$$3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0$$

$$v_C = 10 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 10 \text{ mm/s} \rightarrow \blacktriangleleft$$

(*b*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$$

$$a_C = 6 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_C = 6 \text{ mm/s}^2 \rightarrow \blacktriangleleft$$

Now $$\mathbf{v}_B = \text{constant} \rightarrow a_B = 0$$

Then, substituting into Eq. (2)

$$3a_A + 4(0) + (6 \text{ mm/s}^2) = 0$$

$$a_A = -2 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_A = 2 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

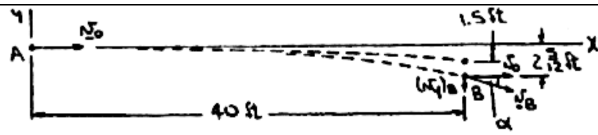
(*c*) We have $$y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

At $t = 5$ s: $$y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2}(-2 \text{ mm/s}^2)(5 \text{ s})^2$$

$$= -175 \text{ mm}$$

or $$\mathbf{y}_A - (\mathbf{y}_A)_0 = 175 \text{ mm} \uparrow \blacktriangleleft$$

Chapter 11, Solution 100



(*a*) Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = v_0 t$$

When $h = 31$ in., $y = -2\frac{5}{12}$ ft: $\quad -2\frac{5}{12}\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$

or $\quad t_{31} = 0.387432$ s

Then $\quad 40\text{ ft} = (v_0)_{31}(0.387432\text{ s})$

or $\quad (v_0)_{31} = 103.244\text{ ft/s} = 70.4\text{ mi/h}$

When $h = 42$ in., $y = -1.5$ ft: $\quad -1.5\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$

or $\quad t_{42} = 0.305234$ s

Then $\quad 40\text{ ft} = (v_0)_{42}(0.305234\text{ s})$

or $\quad (v_0)_{42} = 131.047\text{ ft/s} = 89.4\text{ mi/h}$

$70.4\text{ mi/h} \# \; v_0 \# \; 89.4\text{ mi/h}$ ◀

(*b*) For the vertical motion

$$v_y = (0) - gt$$

Now $\quad \tan\alpha = \frac{|(v_y)_B|}{(v_x)_B} = \frac{gt}{v_0}$

When $h = 31$ in.: $\quad \tan\alpha = \frac{(32.2\text{ ft/s}^2)(0.387432\text{ s})}{103.244\text{ ft/s}}$

$= 0.120833$

or $\quad \alpha_{31} = 6.89°$ ◀

When $h = 42$ in.: $\quad \tan\alpha = \frac{(32.2\text{ ft/s}^2)(0.305234\text{ s})}{131.047\text{ ft/s}}$

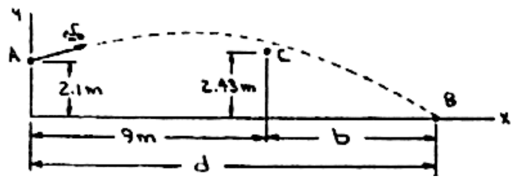
$= 0.075000$

or $\quad \alpha_{42} = 4.29°$ ◀

Chapter 11, Solution 101

First note $(v_x)_0 = (13.40 \text{ m/s}) \cos 20° = 12.5919 \text{ m/s}$

$(v_y)_0 = (13.40 \text{ m/s}) \sin 20° = 4.5831 \text{ m/s}$



(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At C $\quad 9 \text{ m} = (12.5919 \text{ m/s})t \quad \text{or} \quad t_C = 0.71475 \text{ s}$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

At C:

$$y_c = 2.1 \text{ m} + (4.5831 \text{ m/s})(0.71475 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.71475 \text{ s})^2 = 2.87 \text{ m}$$

y_C . 2.43 m (height of net) $\Rightarrow$ ball clears net ◀

(*b*) At B, $y = 0$: $\quad 0 = 2.1 \text{ m} + (4.5831 \text{ m/s})t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$

Solving $\quad t_B = 1.271175 \text{ s}$ (the other root is negative)

Then

$$d = (v_x)_0 t_B = (12.5919 \text{ m/s})(1.271175 \text{ s}) = 16.01 \text{ m}$$

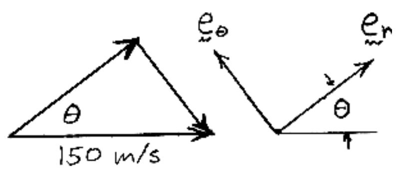
The ball lands $\quad b = (16.01 - 9.00) \text{ m} = 7.01 \text{ m}$ from the net ◀

Chapter 11, Solution 193

Geometry. The polar coordinates are

$$v = \sqrt{(800)^2 + (600)^2} = 1000 \text{ m} \quad \theta = \tan^{-1}\left(\frac{600}{800}\right) = 36.87°$$

Velocity Analysis.



$$\mathbf{v} = 150 \text{ m/s} \rightarrow$$

$$v_r = 150\cos\theta = 120 \text{ m/s}$$

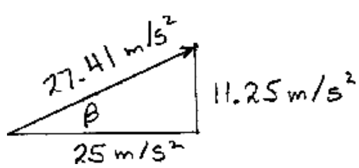
$$v_\theta = -150\sin\theta = -90 \text{ m/s}$$

$$v_r = \dot{r} \qquad \dot{r} = 120 \text{ m/s} \blacktriangleleft$$

$$v_\theta = r\dot{\theta} \quad \dot{\theta} = \frac{v_\theta}{r} = -\frac{90}{1000}$$

$$\dot{\theta} = -0.0900 \text{ rad/s} \blacktriangleleft$$

Acceleration analysis.



$$a_t = 25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(150)^2}{2000} = 11.25 \text{ m/s}^2$$

$$\mathbf{a} = 25 \text{ m/s}^2 \rightarrow + 11.25 \text{ m/s}^2 \uparrow = 27.41 \text{ m/s}^2 \measuredangle\ 24.23°$$

$$\beta = 24.23°$$

$$\theta - \beta = 12.64°$$

$$a_r = a\cos(\theta - \beta) = 27.41\cos 12.64° = 26.74 \text{ m/s}^2$$

$$a_\theta = -a\sin(\theta - \beta) = -27.41\sin 12.64° = -6.00 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \ddot{r} = a_r + r\dot{\theta}^2$$

$$\ddot{r} = 26.74 + (1000)(0.0900)^2 \qquad \ddot{r} = 34.8 \text{ m/s}^2 \blacktriangleleft$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{a_\theta}{r} - \frac{2\dot{r}\dot{\theta}}{r}$$

$$= \frac{-6.00}{1000} - \frac{(2)(120)(-0.0900)}{1000} \qquad \ddot{\theta} = -0.0156 \text{ rad/s}^2 \blacktriangleleft$$