ME 104: Homework 10 Solutions

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Chapter 17, Solution 3

FOR ANY DISK: $m = \rho(\pi r^2 t)$

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

Disk A: $= \frac{1}{2}\pi\rho t r^4 \qquad I_A = \frac{1}{2}\pi\,\rho\,b\,r^4$

Disk B: $I_B = \frac{1}{2}\pi\,\rho(3b)(nr)^4$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B$$

$$= (1 + 3n^4)I_A$$

Work-energy. $T_1 = 0 \quad U_{1\to 2} = M\theta = M(4\pi \text{ rad})$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + M(4\pi) = \frac{1}{2}(1 + 3n^4)I_A\omega_2^2$$

$$\omega_2^2 = \frac{8\pi M}{(1 + 3n^4)I_A}$$

For Point D on rim of disk B

$$v_D = (nr)\omega_2 \quad \text{or} \quad v_D^2 = n^2 r^2 \omega_2^2 = \frac{8\pi M r^2}{I_A}\cdot\frac{n^2}{1 + 3n^4}$$

Value of n for maximum final speed.

For maximum v_D: $\frac{d}{dn}\left(\frac{n^2}{1 + 3n^4}\right) = 0$

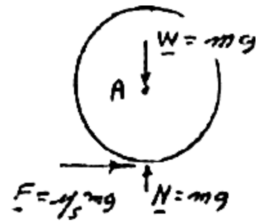
$$\frac{1}{(1 + 3n^4)^2}[n^2(12n^3) - (1 + 3n^4)(2n)] = 0$$

$$12n^5 - 2n - 6n^5 = 0$$

$$2n(3n^4 - 1) = 0$$

$n = 0$ and $n = \left(\frac{1}{3}\right)^{0.25} = 0.7598$ $\qquad n = 0.760$ ◀

Chapter 17, Solution 7



Work of external friction force on disk *A*.

Only force doing work is *F*. Since its moment about *A* is $M = rF$, we have

$$U_{1\to 2} = M\theta$$
$$= rF\theta$$
$$= r(\mu_k mg)\theta$$

Kinetic energy of disk *A*.

Angular velocity becomes constant when $\omega_2 = \frac{v}{r}$

$$T_1 = 0$$
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$
$$= \frac{mv^2}{4}$$

Principle of work and energy for disk *A*.

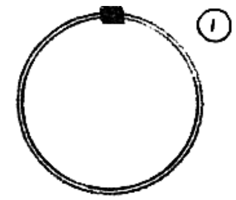
$$T_1 + U_{1\to 2} = T_2: \quad 0 + r\,\mu_k mg\,\theta = \frac{mv^2}{4}$$

Angle change. $\theta = \frac{v^2}{4r\,\mu_k g}$ rad $\qquad \theta = \frac{v^2}{8\pi r \mu_k g}$ rev ◀

Chapter 17, Solution 28

The point of contact with ground is the instantaneous center.

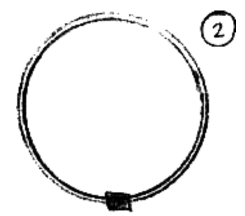
Position 1. Point *B* is at the top.



$$\omega = \omega_1 \quad v_B = 2r\omega_1 \quad v_A = r\omega_1$$

$$T_1 = \frac{1}{2}mv_B^2 + \frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega^2$$
$$= \frac{1}{2}m(2r\omega_1)^2 + \frac{1}{2}m(r\omega_1)^2 + \frac{1}{2}(mr^2)\omega_1^2$$
$$= 3mr^2\omega_1^2$$

Position 2. Point *B* is at the bottom.



$$\omega = \omega_2 \quad v_B = 0 \quad v_A = r\omega_2$$

$$T_2 = \frac{1}{2}mv_B^2 + \frac{1}{2}mv_A^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= 0 + \frac{1}{2}m(r\omega_2)^2 + \frac{1}{2}(mr^2)\omega_2^2$$
$$= mr^2\omega_2^2$$
$$= mr^2(3\omega_1)^2$$
$$= 9mr^2\omega_1^2$$

Work. $$U_{1\to 2} = mg(\Delta h) = 2mgr$$

Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 3mr^2\omega_1^2 + 2mgr = 9m\omega_1^2$$

$$\omega_1^2 = \frac{g}{3r}$$

$$\omega_1 = 0.577\sqrt{\frac{g}{r}} \blacktriangleleft$$

Chapter 17, Solution 55

Mass of disk B.

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A$$

$$= \left(\frac{125\text{ mm}}{100\text{ mm}}\right)^2 3\text{ kg}$$

$$= 4.6875\text{ kg}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}(3\text{ kg})(0.1\text{ m})^2 + \frac{1}{2}(4.6875\text{ kg})(0.125\text{ m})^2$$

$$= 0.05162\text{ kg}\cdot\text{m}^2$$

Angular velocities.

$$\omega_1 = 200\text{ rpm}\left(\frac{2\pi}{60}\right) = 20.944\text{ rad/s}$$

$$\omega_2 = 800\text{ rpm}\left(\frac{2\pi}{60}\right) = 83.776\text{ rad/s}$$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about B: $\bar{I}\omega_1 + Mt = \bar{I}\omega_2$

Couple M.

$$M = \frac{\bar{I}}{t}(\omega_2 - \omega_1)$$

$$= \frac{0.05162\text{ kg}\cdot\text{m}^2}{3\text{ s}}(83.776\text{ rad/s} - 20.944\text{ rad/s})$$

$M = 1.081\text{ N}\cdot\text{m}$ ◀

Chapter 17, Solution 79

Moment of inertia of yoke: $I_C = mk_C^2 = \left(\frac{1.5}{32.2}\right)\left(\frac{3}{12}\right)^2 = 2.9115\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$

Moment of inertia of disk:

$$\theta = 0: \quad I_A = \frac{1}{4}mr^2 = \frac{1}{4}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2 = 2.15666\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°: \quad I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2 = 4.3133\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Total moment of inertia about the *x* axis:

$$\theta = 0: \quad (I_x)_1 = I_C + I_A = 5.0682\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°: \quad (I_x)_2 = I_C + I_A = 7.2248\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Angular momentum about the *x* axis:

$$\theta = 0: \quad H_1 = (I_x)_1\omega_1 = 5.0682\times10^{-3}\omega_1$$

$$\theta = 90°: \quad H_2 = (I_x)_2\omega_2 = 7.2248\times10^{-3}\omega_2$$

Conservation of angular momentum.

$$H_1 = H_2: \quad 5.0682\times10^{-3}\omega_1 = 7.2248\times10^{-3}\omega_2$$

$$\omega_2 = 0.7015\omega_1 = (0.7015)(120\ \text{rpm})$$

$$\omega_2 = 84.2\ \text{rpm} \blacktriangleleft$$

Chapter 17, Solution 85

Let $\boldsymbol{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$

$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $I_C = 650 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

Blades: $I_B = 4\left(\frac{1}{3}mL^2\right) = (4)\left(\frac{1}{3}\right)\left(\frac{55}{32.2}\right)(14)^2$

$$= 446.38 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

The cab does not rotate. $\Omega_1 = \Omega_2 = 0$

$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

Moments about shaft: $I_B(\omega_1 + \Omega_1) + I_C\Omega_1 + Frt = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$

$$Frt = I_B(\omega_2 - \omega_1)$$

$$= (446.38)(8\pi - 6\pi)$$

$$= 2804.7 \text{ lb}\cdot\text{ft}\cdot\text{s}$$

$$Ft = \frac{Frt}{r} = \frac{2804.7}{16} = 175.29 \text{ lb}\cdot\text{s}$$

Linear components: $mv_1 + Ft = mv_2$

$$v_2 - v_1 = \frac{Ft}{m} = \frac{175.29}{\frac{1250}{32.2} + (4)\left(\frac{55}{32.2}\right)}$$

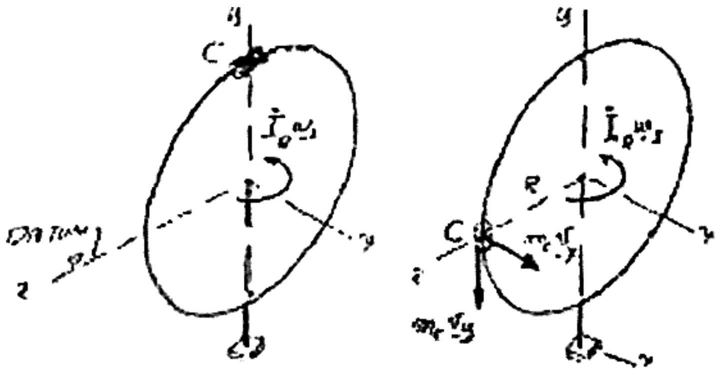
$$= 3.8398 \text{ ft/s}$$

(*a*) Assume $v_1 = 0$. $v_2 = 3.84 \text{ ft/s}$ ◀

(*b*) Force. $F = \frac{Ft}{t} = \frac{175.29}{12}$ $F = 14.61 \text{ lb}$ ◀

Chapter 17, Solution 88

Moment of inertia of ring. $\bar{I}_R = \frac{1}{2} m_R R^2$



Position 1   Position 2

Position 1. $\theta = 0.$

$$v_C = 0$$

Position 2. $\theta = 90°$

$$(v_C)_y = v_y = R\omega_2$$

Conservation of angular momentum about y axis for system.

$$\bar{I}_R \omega_1 = \bar{I}_R \omega_2 + m_C v_y R$$

$$\frac{1}{2} m_R R^2 \omega_1 = \frac{1}{2} m_R R^2 \omega_2 + m_C R^2 \omega_2$$

$$m_R R^2 \omega_1 = (m_R + 2m_C) R^2 \omega_2$$

$$\omega_2 = \frac{m_R}{m_R + 2m_C} \omega_1 \qquad (1)$$

Potential energy. Datum is the center of the ring.

$$V_1 = m_C g R \quad V_2 = 0$$

Kinetic energy: $T_1 = \frac{1}{2} \bar{I}_R \omega_1^2 = \frac{1}{2}\left(\frac{1}{2} m_R R^2\right)\omega_1^2$

$$= \frac{1}{4} m_R R^2 \omega_1^2$$

$$T_2 = \frac{1}{2} \bar{I}_R \omega_2^2 + \frac{1}{2} m_C \left(v_x^2 + v_y^2\right)$$

$$= \frac{1}{4} m_R R^2 \omega_2^2 + \frac{1}{2} m_C R^2 v_2^2 + \frac{1}{2} m_C v_y^2$$

PROBLEM 17.88 (Continued)

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{4}m_R R^2\omega_1^2 + m_C gR = \left(\frac{1}{4}m_R + \frac{1}{2}m_C\right)R^2\omega_2^2 + \frac{1}{2}m_C v_y^2 \qquad (2)$$

Data:

$$m_C = 2 \text{ kg}$$
$$m_R = 3 \text{ kg}$$
$$R = 0.25 \text{ m}$$
$$\omega_1 = 35 \text{ rad/s}$$

(*a*) Angular velocity.

From Eq. (1),
$$\omega_2 = \frac{3 \text{ kg}}{3 \text{ kg} + 2(2 \text{ kg})}(35 \text{ rad/s})$$

$\omega_2 = 15.00$ rad/s ◀

(*b*) Velocity of collar relative to ring.

From Eq. (2),
$$\frac{1}{4}(3 \text{ kg})(0.25 \text{ m})^2(35 \text{ rad/s})^2 + (2 \text{ kg})(9.81 \text{ m/s}^2)(0.25 \text{ m})$$
$$= \left[\frac{1}{4}(3 \text{ kg}) + \frac{1}{2}(2 \text{ kg})\right](0.25 \text{ m})^2(15 \text{ rad/s})^2 + \frac{1}{2}(2 \text{ kg})v_y^2$$
$$57.422 + 4.905 = 24.609 + v_y^2$$
$$v_y^2 = 37.716$$

$v_y = 6.14$ m/s ◀

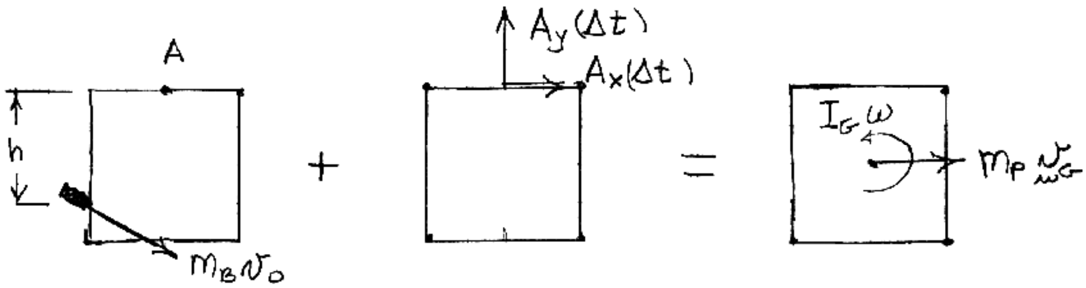
Chapter 17, Solution 98

$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6} m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg} \cdot \text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point A with angular velocity $\boldsymbol{\omega} = \omega$ ↻.

$$\mathbf{v}_G = \frac{b}{2}\omega \longrightarrow$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

(a) +↻ Moments about A:

$$(m_B v_0 \cos 30°)h + m_B v_0 \sin 30° \left(\frac{b}{2}\right) + 0 = I_G\omega + m_P v_G \frac{b}{2}$$

$$m_B v_0 \left(h\cos 30° + \frac{b}{2}\sin 30°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(0.150\cos 30° + 0.100\sin 30°)$$

$$= \left[0.06 + \frac{1}{4}(9)(0.2)^2\right]\omega = 0.15\omega$$

$$\omega = 21.588 \text{ rad/s}$$

$$v_B = (0.100)(21.556) = 2.1556 \text{ m/s} \qquad \mathbf{v}_G = 2.16 \text{ m/s} \longrightarrow \blacktriangleleft$$

(b) $\xrightarrow{+}$ Linear momentum:

$$m_B v_0 \cos 30° + A_x(\Delta t) = m_P v_G$$

$$(0.045)(400\cos 30°) + A_x(0.002) = (9)(2.1556)$$

$$\mathbf{A}_X = 1920 \text{ N} \qquad \mathbf{A}_x = 1920 \text{ N} \longrightarrow$$

+↑ Linear momentum:

$$-m_B v_0 \sin 30° + A_y(\Delta t) = 0$$

$$-(0.045)(400)\sin 30° + A_y(0.002) = 0$$

$$A_y = 4500 \text{ N} \qquad \mathbf{A}_y = 4500 \text{ N} \uparrow$$

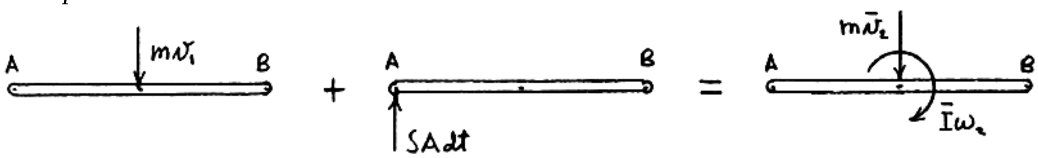
$$A = 4892 \text{ N} = 4.892 \text{ kW} \qquad \tan\beta = \frac{4500}{1920} \qquad \beta = 66.9°$$

$$\mathbf{A} = 4.87 \text{ kN} \angle 66.9° \blacktriangleleft$$

Chapter 17, Solution 106

Moment of inertia. $\bar{I} = \frac{1}{12} mL^2$

(*a*) *First Impact at A.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Condition of impact: $e = 1: \quad (\mathbf{v}_A)_2 = v_1 \uparrow$

Kinematics: $\bar{v}_2 = \frac{L}{2}\omega - (v_A)_2 = \frac{L}{2}\omega - v_1$

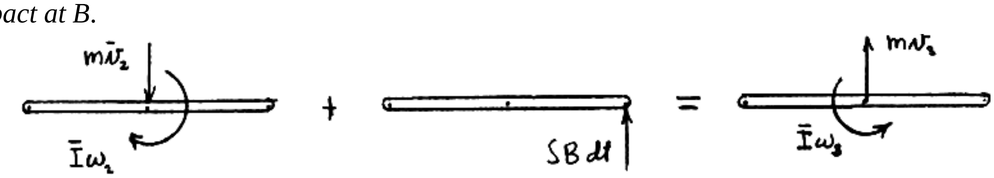
↻ Moments about A: $mv_1 \frac{L}{2} + 0 = m\bar{v}_2 \frac{L}{2} + \bar{I}\omega_2$

$$= m\left(\frac{L}{2}\omega - v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_2 \qquad \omega_2 = \frac{3v_1}{L} \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{L}{2}\left(\frac{3v_1}{L}\right) - v_1 = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{2}v_1 \downarrow \blacktriangleleft$$

$$(v_B)_2 = L\omega - (v_A)_2 = 3v_1 - v_1 = 2v_1 \downarrow$$

(*b*) *Impact at B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_3$

Condition of impact. $e = 1: \quad (\mathbf{v}_B)_3 = 2v_1 \uparrow$

Kinematics: $\bar{v}_3 = (v_B)_2 - \frac{L}{2}\omega = 2v_1 - \frac{L}{2}\omega$

↺ Moments about B: $-m\bar{v}_2 \frac{L}{2} + \bar{I}\omega_2 + 0 = m\bar{v}_3 \frac{L}{2} - \bar{I}\omega_3$

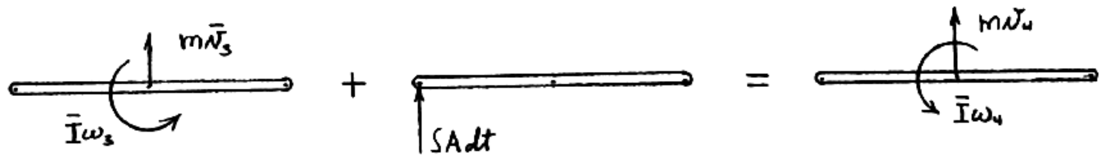
$$-m\left(\frac{1}{2}v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right) + 0 = m\left(2v_1 - \frac{L}{2}\omega_3\right)\frac{L}{2} - \left(\frac{1}{12}mL^2\right)\omega_3 \qquad \omega_3 = \frac{3v_1}{L} \circlearrowleft \blacktriangleleft$$

$$\bar{v}_3 = 2v_1 - \frac{L}{2}\left(\frac{3v_1}{L}\right) = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_3 = \frac{1}{2}v_1 \uparrow \blacktriangleleft$$

$$(v_A)_3 = L\omega - (v_B)_3 = 3v_1 - 2v_1 = v_1 \downarrow$$

PROBLEM 17.106 (Continued)

(*c*) *Second Impact at A.*



Syst. Momenta$_3$ + **Syst. Ext. Imp.**$_{3\to4}$ = **Syst. Momenta**$_4$

Condition of impact. $e = 1$: $(\mathbf{v}_A)_4 = v_1 \uparrow$

Kinematics: $\bar{v}_4 = (v_A)_4 + \dfrac{L}{2}\omega_4 = v_1 + \dfrac{L}{2}\omega_4$

↺ Moments about *A*: $m\bar{v}_3 \dfrac{L}{2} + \bar{I}\omega_3 + 0 = m\bar{v}_4 \dfrac{L}{2} + \bar{I}\omega_4$

$$m\left(\frac{1}{2}v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right) + 0 = m\left(v_1 + \frac{L}{2}\omega_4\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_4$$

$\boldsymbol{\omega}_4 = 0$ ◀

$$\bar{v}_4 = v_1 + 0$$

$\bar{\mathbf{v}}_4 = v_1 \uparrow$ ◀