ME 104: Homework 1 Solutions

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Chapter 11, Solution 2

Given:

$$x = 12t^3 - 18t^2 + 2t + 5$$

$$v = \frac{dx}{dt} = 36t^2 - 36t + 2$$

$$a = \frac{dv}{dt} = 72t - 36$$

Find the time for $a = 0$.

$$72t - 36 = 0 \Rightarrow t = 0.5 \text{ s}$$

Substitute into above expressions.

$$x = 12(0.5)^3 - 18(0.5)^2 + 2(0.5) + 5 = 3 \qquad x = 3.00 \text{ m} \blacktriangleleft$$

$$v = 36(0.5)^2 - 36(0.5) + 2$$
$$= -7 \text{ m/s} \qquad v = -7.00 \text{ m/s} \blacktriangleleft$$

Chapter 11 Solution 4

We have	$x = 6t^2 - 8 + 40\cos \pi t$		
Then	$v = \dfrac{dx}{dt} = 12t - 40\pi \sin \pi t$		
and	$a = \dfrac{dv}{dt} = 12 - 40\pi^2 \cos \pi t$		
At $t = 6$ s:	$x_6 = 6(6)^2 - 8 + 40\cos 6\pi$	or	$x_6 = 248$ in. ◀
	$v_6 = 12(6) - 40\pi \sin 6\pi$	or	$v_6 = 72.0$ in./s ◀
	$a_6 = 12 - 40\pi^2 \cos 6\pi$	or	$a_6 = -383$ in./s^2 ◀

Chapter 11, Solution 7

We have $$x = t^3 - 6t^2 - 36t - 40$$

Then $$v = \frac{dx}{dt} = 3t^2 - 12t - 36$$

and $$a = \frac{dv}{dt} = 6t - 12$$

(*a*) When $v = 0$: $$3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$$

or $$(t+2)(t-6) = 0$$

or $t = -2$ s (Reject) and $t = 6$ s $\qquad t = 6.00 \text{ s}$ ◀

(*b*) When $x = 0$: $$t^3 - 6t^2 - 36t - 40 = 0$$

Factoring $(t-10)(t+2)(t+2) = 0$ or $t = 10$ s

Now observe that $0 \le t < 6$ s: $v < 0$

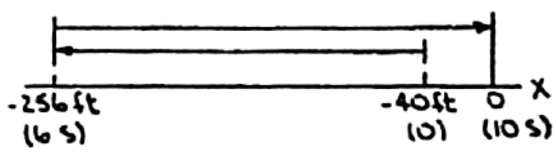
$6 \text{ s} < t \le 10$ s: $v > 0$

and at $t = 0$: $x_0 = -40$ ft

$t = 6$ s: $$x_6 = (6)^3 - 6(6)^2 - 36(6) - 40$$
$$= -256 \text{ ft}$$

$t = 10$ s: $v_{10} = 3(10)^2 - 12(10) - 36$ $\qquad$ or $v_{10} = 144.0$ ft/s ◀

$a_{10} = 6(10) - 12$ $\qquad$ or $a_0 = 48.0 \text{ ft/s}^2$ ◀



Then $$|x_6 - x_0| = |-256 - (-40)| = 216 \text{ ft}$$
$$x_{10} - x_6 = 0 - (-256) = 256 \text{ ft}$$

Total distance traveled $= (216 + 256)$ ft $= 472$ ft ◀

Chapter 11, Solution 10

We have $a = kt^2 \quad k = \text{constant}$

Now $\dfrac{dv}{dt} = a = kt^2$

At $t = 6$ s, $v = 18$ m/s: $\displaystyle\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$

or $v - 18 = \dfrac{1}{3}k(t^3 - 216)$

or $v = 18 + \dfrac{1}{3}k(t^3 - 216)(\text{m/s})$

Also $\dfrac{dx}{dt} = v = 18 + \dfrac{1}{3}k(t^3 - 216)$

At $t = 0$, $x = 24$ m: $\displaystyle\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right]dt$

or $x - 24 = 18t + \dfrac{1}{3}k\left(\dfrac{1}{4}t^4 - 216t\right)$

Now

At $t = 6$ s, $x = 96$ m: $96 - 24 = 18(6) + \dfrac{1}{3}k\left[\dfrac{1}{4}(6)^4 - 216(6)\right]$

or $k = \dfrac{1}{9}\ \text{m/s}^4$

Then $x - 24 = 18t + \dfrac{1}{3}\left(\dfrac{1}{9}\right)\left(\dfrac{1}{4}t^4 - 216t\right)$

or $x(t) = \dfrac{1}{108}t^4 + 10t + 24$ ◀

and $v = 18 + \dfrac{1}{3}\left(\dfrac{1}{9}\right)(t^3 - 216)$

or $v(t) = \dfrac{1}{27}t^3 + 10$ ◀

Chapter 11, Solution 24

$$v_0 = 25 \text{ ft/s}, \quad x - x_0 = 30 \text{ ft}$$

$$a = 10 - 0.9v^2 = k(c^2 - v^2)$$

Where $k = 0.9 \text{ ft}^{-1}$ and $c^2 = \dfrac{10}{0.9} = 11.111 \text{ ft}^2/\text{s}^2$

$$c = 3.3333 \text{ ft/s}$$

Since $v_0 .\ \ c,$ write

$$a = v\frac{dv}{dx} = -k(v^2 - c^2)$$

$$\frac{v\,dv}{v^2 - c^2} = -k\,dx$$

Integrating, $\dfrac{1}{2}\ln(v^2 - c^2)\Big|_{v_0}^{v} = -k(x - x_0)$

$$\ln\frac{v^2 - c^2}{v_0^2 - c^2} = -2k(x - x_0)$$

$$\frac{v^2 - c^2}{v_0^2 - c^2} = e^{-2k(x - x_0)}$$

$$v^2 = c^2 + \left(v_0^2 - c^2\right)e^{-2k(x - x_0)}$$

$$= 11.111 + [(25)^2 - 11.111]\, e^{-(2)(0.9)(30)}$$

$$= 11.111 + 3.89 \times 10^{-19} = 11.111 \text{ ft}^2/\text{s}^2$$

$v = 3.33 \text{ ft/s}$ ◀

Chapter 11, Solution 28

(*a*) We have

$$a = v\frac{dv}{dx}$$

$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$

$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or $a = -0.0525\ \text{m/s}^2$ ◀

(*b*) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x\,dx = \int_{t_1}^{t_3} 0.18v_0\,dt$$

or

$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or

$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or $t_3 - t_1 = 6.17$ s ◀

Chapter 11, Solution 29

We have $$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1+\frac{4}{20.9\times 10^6}\right)^2}$$

When $$y = 0, \qquad v = v_0$$
$$y = y_{\max}, \quad v = 0$$

Then $$\int_{v_0}^{0} v\,dv = \int_0^{y_{\max}} \frac{-32.2}{\left(1+\frac{y}{20.9\times 10^6}\right)^2}dy$$

or $$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times 10^6 \frac{1}{1+\frac{4}{20.9\times 10^6}}\right]_0^{y_{\max}}$$

or $$v_0^2 = 1345.96\times 10^6\left(1-\frac{1}{1+\frac{y_{\max}}{20.9\times 10^6}}\right)$$

or $$y_{\max} = \frac{v_0^2}{64.4-\frac{v_0^2}{20.9\times 10^6}}$$

(a) $v_0 = 1800$ ft/s: $$y_{\max} = \frac{(1800)^2}{64.4-\frac{(1800)^2}{20.9\times 10^6}}$$

or $$y_{\max} = 50.4\times 10^3 \text{ ft} \blacktriangleleft$$

(b) $v_0 = 3000$ ft/s: $$y_{\max} = \frac{(3000)^2}{64.4-\frac{(3000)^2}{20.9\times 10^6}}$$

or $$y_{\max} = 140.7\times 10^3 \text{ ft} \blacktriangleleft$$

(c) $v_0 = 36,700$ ft/s: $$y_{\max} = \frac{(36,700)^2}{64.4-\frac{(36,700)^2}{20.9\times 10^6}}$$

or $$y_{\max} = -3.03\times 10^{10} \text{ ft} \blacktriangleleft$$

The velocity 36,700 ft/s is approximately the escape velocity v_R from the earth. For v_R

$$y_{\max} \to \infty \blacktriangleleft$$

Chapter 11, Solution 30

We have $$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When $$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

Then $$\int_{v_e}^{0} v\,dv = \int_{R}^{\infty} -\frac{gR^2}{r^2}\,dr$$

or $$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^{\infty}$$

or $$v_e = \sqrt{2gR}$$
$$= \left(2 \times 32.2 \text{ ft/s}^2 \times 3960 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}}\right)^{1/2}$$

or $$v_e = 36.7 \times 10^3 \text{ ft/s} \blacktriangleleft$$

Chapter 11, Solution 183

We have $$v\frac{dv}{dx} = a = -60x^{-1.5}$$

When $x = 4$ m, $v = 0$: $$\int_0^v v\,dv = \int_4^x (-60x^{-1.5})dx$$

or $$\frac{1}{2}v^2 = 120[x^{-0.5}]_4^x$$

or $$v^2 = 240\left(\frac{1}{\sqrt{x}} - \frac{1}{2}\right)$$

(*a*) When $x = 2$ m: $$v^2 = 240\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

or $v = -7.05$ m/s ◀

(*b*) When $x = 1$ m: $$v^2 = 240\left(1 - \frac{1}{2}\right)$$

or $v = -10.95$ m/s ◀

(*c*) When $x = 0.1$ m: $$v^2 = 240\left(\frac{1}{\sqrt{0.1}} - \frac{1}{2}\right)$$

or $v = -25.3$ m/s ◀

Chapter 11, Solution 184

First note

When $x = \frac{4}{12}$ ft, $v = 0$: $\quad 0 = (900 \text{ ft/s}) - k\left(\frac{4}{12} \text{ ft}\right)$

or $\quad k = 2700 \frac{1}{\text{s}}$

(*a*) We have $\quad v = v_0 - kx$

Then $\quad a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$

or $\quad a = -k(v_0 - kx)$

At $t = 0$: $\quad a = 2700 \frac{1}{\text{s}}(900 \text{ ft/s} - 0)$

or $\quad a_0 = -2.43 \times 10^6 \text{ ft/s}^2$ ◀

(*b*) We have $\quad \frac{dx}{dt} = v = v_0 - kx$

At $t = 0, x = 0$: $\quad \int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$

or $\quad -\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$

or $\quad t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$

When $x = 3.9$ in.: $\quad t = \frac{1}{2700 \frac{1}{\text{s}}}\ln\left[\frac{1}{1 - \frac{2700 \text{ 1/s}}{900 \text{ ft/s}}\left(\frac{3.9}{2} \text{ ft}\right)}\right]$

or $\quad t = 1.366 \times 10^{-3} \text{ s}$ ◀