

REAL-TIME ESTIMATION OF INTERSECTION TURNING PROPORTIONS FROM EXIT COUNTS

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Abstract

1
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3 Estimation of intersection origin-destination (OD) matrices is a frequently-encountered problem in
4 transportation management operations. In particular, advanced traffic responsive arterial signalization
5 management schemes rely on the accurate real-time estimation of vehicle queue lengths and turning ratios.
6 This paper proposes a new method for estimating OD matrices of a signalized intersection in real time.
7 The proposed method only requires detectors at exit legs, saving half detectors from a complete
8 configuration assumed by previous research. Signal phase timing information is used to determine the
9 portion of the volume measurement that corresponds to a specific phase. The estimation of turning
10 proportions is formulated as a constrained least squares problem and solved using recursive algorithm
11 with forgetting factor and covariance resetting. Simulation shows that the proposed method can obtain
12 accurate results and response to parameter changes fast, while requiring significant less computational
13 effort than a comparable solution using a constrained optimization method.

1 I - INTRODUCTION

2
3 In advanced intersection signal operations, one frequent requirement is the real-time knowledge of turning
4 volumes, or turning proportions (1). This information is usually described by the so-call origin-destination
5 (OD) matrices. In the past decades, great attention has been received in estimating OD matrices from
6 traffic counts collected by sensors, and a lot of methods have been proposed (2).

7 In most of the previous research, a complete configuration of detectors was assumed to be
8 available. A complete configuration means detectors are installed at all entrance legs and exit legs for
9 measuring traffic volumes. In such a configuration, the problem of estimating turning proportions could
10 be formulated as a linear system with linear equality and inequality constraints. This system is
11 underdetermined, and least squares method could be used to solve it. To solve a constrained least squares
12 problem, people can use a variety of constrained quadratic optimization algorithms, or routine in (3).
13 However, a direct solution of constrained least squares problems through optimization could be
14 computationally expensive, thus inappropriate for real-time practice. With this concern, different
15 recursive approaches have been proposed. Early Examples can be found in (4, 5). Those recursive
16 methods usually included two major steps. In the first step, unconstrained least squares estimator is used.
17 In the second step, parameter estimates are constrained using some correction methods, such as
18 normalization and truncation. Because of the operation of normalization or truncation, there is no
19 guarantee that the resulting estimates are optimal. Motivated by this concern, (6) developed a simple
20 algorithm for the least squares problem with inequality boundary constraints. Unlike the previous
21 methods, constraints were enforced by an iteration process in this algorithm. The convergence of this
22 algorithm was proven, and it was stated that the imposition of the inequality constraints in the update
23 iteration could improve the accuracy. Based on this algorithm, (7) proposed a recursive method which
24 further incorporated both equality and boundary inequality constraints in the least squares update. This
25 method could achieve good estimation in much smaller computational effort, compared to an optimization
26 approach.

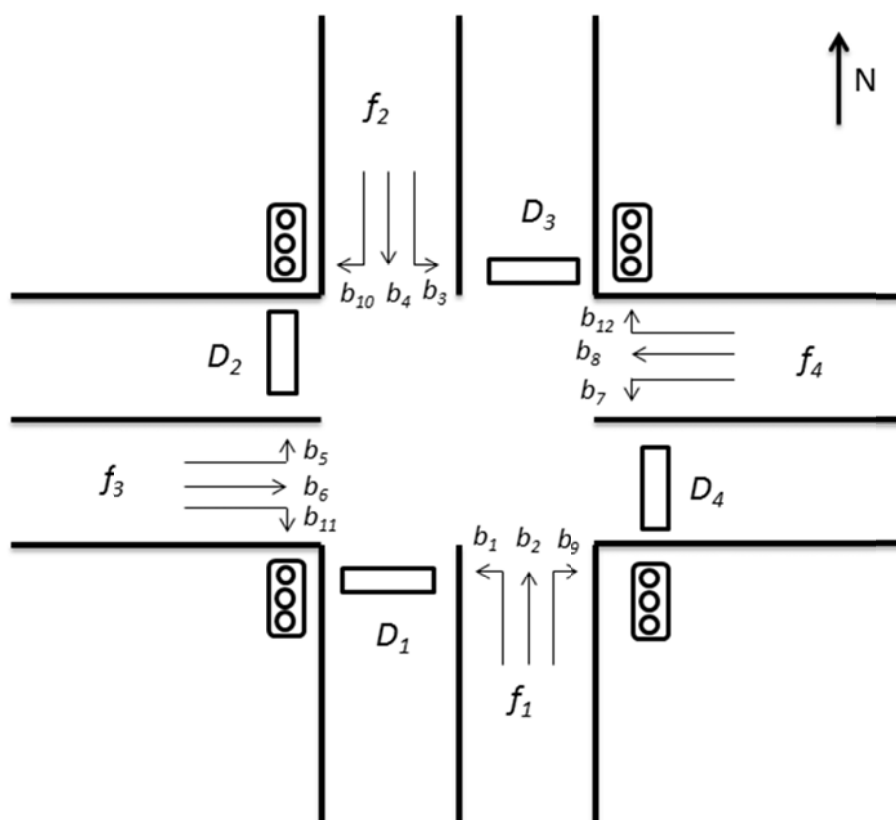
27 Even though recursive approaches have been shown to be computationally efficient and well-
28 performed, there are still two concerns about these methods. The first concern is that a complete
29 configuration of detectors may not be available. In a complete configuration, a typical four-way
30 intersection requires at least eight detectors, whose price might be expensive. In order to get an economic
31 configuration, a “Cordon” configuration, which groups several intersections and deletes one or more
32 intermediate detectors, can be used. (8) firstly introduced a condition to examine the feasibility of
33 estimating from an incomplete set of traffic counts. As long as this condition is met, it is able to obtain
34 estimates from the incomplete information. But it was also mentioned in (8) that an increase in variability
35 could happened since less traffic counts were accessible. Subsequently, (9) suggested that the condition
36 number of the Jacobian matrix, which is derived from predicted output counts with respect to the
37 parameters, could be used to evaluate whether the incomplete information was sufficient to provide “good”
38 estimates. In its numerical test, it was shown that the condition number enlarged dramatically as detector
39 saving grew, which might lead to larger potential of poor estimation. Some real-time estimation strategies
40 addressing incomplete detector configuration can be found in (10, 11, 12).

41 The second concern of some previously proposed methods is that they might fail to detect
42 changes in parameters. Most of the recursive methods were designed to estimate dynamic turning
43 proportions in real time. To achieve this, time-series of data were used. However, most of those methods,
44 including some mentioned above, were only evaluated with static turning proportions in the paper they
45 presented. Performance of these methods was not shown for the case that turning proportions changed
46 during simulation. Some research has shown that the recursive least squares estimator parameter
47 adaptation gain decreases as time progresses. Thus, an ordinary least squares estimator may fail to follow
48 parameter changes in a dynamic scenario. The technique of forgetting factor or covariance resetting has
49 been suggested by researchers for tracking time-varying parameters. Description of these techniques can
50 be found in (13, 14).

1 This paper presents a new method for estimating turning proportions in real time. Compared with
 2 previous research, the algorithm proposed in this paper only requires detectors to be installed at exit legs.
 3 Hence, it saves half detectors from a complete detector configuration. However, real-time signal is used to
 4 relate the portion of traffic volumes to its corresponding phase. The estimation problem is formulated as a
 5 constrained least squares problem, and then solved recursively. Therefore, it is called Recursive
 6 Constrained Least Squares (RCLS). An extended version of RCLS is presented for the case of time-
 7 varying turning proportions. It applies forgetting factor and covariance resetting. This extended version is
 8 called Recursive Constrained Least Squares with Forgetting Factor and Covariance Resetting (RCLSFR).

9 The remainder of the paper is organized as follows: Section II describes the mathematical
 10 formulation of the estimation problem. Section III explains the steps used to solve the problem. Section
 11 IV discusses how to introduce a forgetting factor and covariance resetting in the parameter adaptation
 12 algorithm. Section V evaluates the algorithm using simulation. Conclusion is presented in Section VI.

14 II - PROBLEM FORMULATION



16
 17
 18 **FIGURE 1 Intersection with departure detectors.**

19
 20 Fig. 1 shows a four-way signalized intersection with detectors presented at exit legs. The northbound and
 21 southbound arrival volumes are denoted as f_1 and f_2 . b_1 and b_2 are used to represent the northbound left
 22 and through percentages, and b_3 and b_4 to represent the southbound left and through percentages. As a
 23 consequence, the northbound and southbound right-turn percentages, denoted as b_9 and b_{10} , equal to $1-b_1-$
 24 b_2 and $1-b_3-b_4$, respectively. Similar notations are used for eastbound and westbound. Assume that there is

Su, Horowitz, Lu

1 no right-turn on red, and the measurements are accurate. During the period that northbound traffic and
 2 southbound traffic get green, the numbers of vehicles passing each detector are denoted as D_1 to D_4 . From
 3 flow conservation, it is straightforward to obtain

$$4 \quad f_2 b_4 = D_1 \quad (1)$$

$$f_1 b_1 + f_2 (1 - b_3 - b_4) = D_2 \quad (2)$$

$$f_1 b_2 = D_3 \quad (3)$$

$$f_1 (1 - b_1 - b_2) + f_2 b_3 = D_4 \quad (4)$$

5
 6 In the period that eastbound traffic and westbound traffic get green, another set of traffic counts
 7 should be collected. One can easily obtain the same equations, only that the subscripts will change.
 8 Therefore, the ideas discussed below apply to both north-south-bound traffic and east-west-bound traffic,
 9 with the only difference subscripts. Hence, in the discussion that follows, the north-south-bound traffic
 10 will be taken as a representative and east-west-bound traffic will not be discuss further.

11 Notice that no assumption is made regarding the lane configuration of intersections, or the type of
 12 signal control. Having a designated lane for a particular movement or not, or being a pre-time control or
 13 actuated control, will not change equation (1) - (4). However, it is necessary for north-south-bound and
 14 east-west-bound traffic not to receive green at the same time, which is frequently the case with most
 15 signalized intersections.

16 Using (1) and (3) to substitute f_1 and f_2 in (2) and (4), one obtains

$$17 \quad D_3 \frac{b_1}{b_2} + D_1 \left(\frac{1}{b_4} - \frac{b_3}{b_4} - 1 \right) = D_2 \quad (5)$$

$$D_3 \left(\frac{1}{b_2} - \frac{b_1}{b_2} - 1 \right) + D_1 \frac{b_3}{b_4} = D_4 \quad (6)$$

18
 19 Because all the b_i 's must fall in the interval of $[0, 1]$, one have the constraints

$$20 \quad \frac{1}{b_2}, \frac{1}{b_4} \geq 1 \quad (7)$$

$$\frac{1}{b_2} \geq \frac{b_1}{b_2} \geq 0 \quad (8)$$

$$\frac{1}{b_4} \geq \frac{b_3}{b_4} \geq 0 \quad (9)$$

21
 22 Also, because $b_1 + b_2 \leq 1$ and $b_3 + b_4 \leq 1$, there are another two constraints

$$23 \quad \frac{b_1}{b_2} - \frac{1}{b_2} \leq -1 \quad (10)$$

$$\frac{b_3}{b_4} - \frac{1}{b_4} \leq -1 \quad (11)$$

24
 25 Constraints (7) - (11) can be rewritten as

26

Su, Horowitz, Lu

$$\frac{1}{b_2}, \frac{1}{b_4} \geq 1 \quad (12)$$

$$\frac{b_1}{b_2}, \frac{b_3}{b_4} \geq 0 \quad (13)$$

$$\frac{1}{b_2} - \frac{b_1}{b_2} \geq 1 \quad (14)$$

$$\frac{1}{b_4} - \frac{b_3}{b_4} \geq 1 \quad (15)$$

1
2 Equation (5), (6) and constraints (12) - (15) define a problem for b_1 to b_4 . There are four unknowns, two
3 equations, and six inequalities. This is an underdetermined problem. It can be solved in the least squares
4 sense.

5 Since (5), (6), (12) - (15) are not in a good form to apply least squares estimator, they are
6 reorganized into a matrix form (16) and (17).
7

$$\begin{bmatrix} 0 & D_3 & D_1 & -D_1 \\ D_3 & -D_3 & 0 & -D_1 \end{bmatrix} \begin{bmatrix} \frac{1}{b_2} - 1 \\ \frac{b_1}{b_2} \\ \frac{1}{b_4} - 1 \\ \frac{b_3}{b_4} \end{bmatrix} = \begin{bmatrix} D_2 \\ D_4 \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \frac{1}{b_2} - 1 \\ \frac{b_1}{b_2} \\ \frac{1}{b_4} - 1 \\ \frac{b_3}{b_4} \end{bmatrix} \geq 0 \quad (17)$$

8
9 Let $\beta = \left[\frac{1}{b_2} - 1, \frac{b_1}{b_2}, \frac{1}{b_4} - 1, \frac{b_3}{b_4} \right]^T$, $X = \begin{bmatrix} 0 & D_3 & D_1 & -D_1 \\ D_3 & -D_3 & 0 & -D_1 \end{bmatrix}$ and $Y = \begin{bmatrix} D_2 \\ D_4 \end{bmatrix}$, the problem (16) and
10 (17) can be represented by
11

$$Y = X\beta \quad (18)$$

12 Subject to

$$\beta \geq 0 \quad (19)$$

15
16 In practice, time-series of data are used. Hence, it is necessary to add time index in equation (18).
17 In addition, sensor noise, vehicle traverse delay and right-turn-on-red vehicles will make detector

Su, Horowitz, Lu

1 measurements not as ideal as in the assumption. The influences of these factors can be represented by an
2 error term in the formulation. As a result, a more exact expression of equation (18) should be

$$Y(k) = X(k)\beta + v(k) \quad (20)$$

3
4 Here k represents time index and v represents measurement error. To solve for β in the least squares
5 sense, one minimizes the object function (21).
6

$$\min_{\hat{\beta}} \sum_{k=1}^N (Y(k) - X(k)\hat{\beta})^T (Y(k) - X(k)\hat{\beta}) \quad (21)$$

7
8 The hat on the variable denotes that it is an estimated value. (19) - (21) define a least squares problem
9 with inequality boundary constraints. It can be solved by an algorithm proposed by Bell (6) and Li (7).
10 Notice that constraints (10) and (11) are excluded in (17), because it will bring in linear constraints other
11 than boundary constraints, and consequently makes computation difficult. The problem will be solved in
12 the following procedure. In the first step, an ordinary least squares estimator is used to solve (20). In the
13 second step, Bell's correction developed in (6) will be applied if constraint (19) is not met. In the third
14 step, \hat{b}_i 's are solved from $\hat{\beta}$. Within this step, normalization and truncation will be conducted if
15 constraints (10) and (11) are not satisfied. Details of the solving procedure can be found in next section.
16

17 III - STEPS TO SOLVE THE LEAST SQUARES PROBLEM

18
19 **Step 0:** Initialize $\hat{\beta}_0$ and P_0 . $\hat{\beta}_0$ is a 4×1 nonnegative vector. It can be initialized by prior knowledge, or
20 random values. P_0 is a 4×4 positive matrix.

21 **Step 1:** Update by an ordinary least squares estimator. After collecting new traffic counts, construct X_k
22 and Y_k , and obtain the estimates $\hat{\beta}_k$ by

$$\hat{\beta}_k = \hat{\beta}_{k-1} + K_k (Y_k - X_k \hat{\beta}_{k-1}) \quad (22)$$

24
25 Where
26

$$K_k = P_{k-1} X_k^T S_k^{-1} \quad (23)$$

$$S_k = X_k P_{k-1} X_k^T + I \quad (24)$$

$$P_k = (I - K_k X_k) P_{k-1} \quad (25)$$

27
28 Here I is the identity matrix.

29 **Step 2:** If there is any negative value in $\hat{\beta}_k$, apply Bell's correction developed in (6) to guarantee the
30 nonnegative constraint is met. Otherwise, skip this step.

31 2.1 Let $\tilde{\beta}_k = \hat{\beta}_k$ and $\mu = 0$.

32 2.2 Compute $D = \text{diag}\{1/p_{ii}\}$, where $P_k = [p_{ij}]_{4 \times 4}$.

33 2.3 Compute $\mu = TF(\mu - D\tilde{\beta}_k)$. Here $TF(\bullet)$ is a truncation function which drives all negative
34 values to zeros.

35 2.4 $\tilde{\beta}_k = \hat{\beta}_k + P_k \mu$.

36 Repeat 2.3 and 2.4 until $\tilde{\beta}_k$ converges. If step 2 is executed, $\hat{\beta}_k$ will be replaced by $\tilde{\beta}_k$.

Su, Horowitz, Lu

1 **Step 3:** Solve for \hat{b}_i 's from $\hat{\beta}_k$. If there is any \hat{b}_i being negative or greater than one, truncate it to zero
 2 or one. If $\hat{b}_1 + \hat{b}_2 > 1$, normalize \hat{b}_1 and \hat{b}_2 so that $\hat{b}_1 + \hat{b}_2 = 1$. Normalization is done by setting
 3 $\hat{b}_1 = \frac{\hat{b}_1}{(\hat{b}_1 + \hat{b}_2)}$ and $\hat{b}_2 = \frac{\hat{b}_2}{(\hat{b}_1 + \hat{b}_2)}$. Apply normalization if $\hat{b}_3 + \hat{b}_4 > 1$. Compute the turning proportions for
 4 right-turns \hat{b}_9 and \hat{b}_{10} by $1 - \hat{b}_1 - \hat{b}_2$ and $1 - \hat{b}_3 - \hat{b}_4$, respectively.

5
 6 The above steps, Step 0 to Step 3, are the solving procedure in RCLS. Step 1 to step 3 are
 7 repeated if new measurements are available.

8 VI - INCORPORATE FORGETTING FACTOR AND COVARIANCE RESETTING

9
 10 Because the matrix S_k in equation (24) keeps growing, the gain (K_k in equation (22)) in an ordinary
 11 least squares estimator would become smaller and smaller as the number of steps grows. Hence an
 12 ordinary least squares estimator might fail to trace time-varying variables. If the turning proportions
 13 change during a studied period, the extended version of RCLS, RCLSFR, should be used. In RCLSFR,
 14 forgetting factor and covariance resetting proposed in (13, 14) are applied in the covariance update. In this
 15 case, equation (26) is used instead of (25) in Step 1.

$$16 P_k = \frac{1}{\lambda} (I - K_k X_k) P_{k-1} + \varepsilon I - \delta P_{k-1}^2 \quad (26)$$

17
 18 In (26), λ is the forgetting factor, and it usually falls in [0.9, 1]. ε and δ are small values to adjust
 19 covariance resetting. They are in [0, 0.1] in usual practice.

20 Notice that, in the case that equation (26) is used in Step 1, the matrix P_k in Step 2 should still be
 21 updated through equation (23) - (25). As a result, two different sets of matrices K_k , S_k , and P_k have to
 22 be maintained during the solving procedure.

23 V - SIMULATION AND RESULTS

24
 25 In this section, the accuracy and computational effort of the proposed method will be evaluated by
 26 simulation. The estimated turning proportions from the proposed approach, RCLS or RCLSFR, and those
 27 from an optimization approach will be compared. The optimization approach chosen is the Constrained
 28 Least Squares by Active-Set algorithm (CLSAS) in Matlab 2013a. Two scenarios, one with static turning
 29 proportions and the other with time-varying proportions, will be simulated. In each scenario, each tested
 30 method will have 10 simulation runs with different random seeds.

31 Scenario I

32
 33 In this scenario, static turning proportions will be used to compare RCLS and CLSAS. Consider a four-
 34 way intersection with turning proportions shown in Table 1 (source from (11)). Arrival volumes are
 35 random variables generated from Poisson distribution with mean 100. Turning volumes are multinomial
 36 random numbers generated from arrival volumes and turning proportions define in Table 1. Exit volume
 37 at each exit leg would be the sum of all turning volumes entering that leg, denoted as q_i . The measurement
 38 error is modeled as Gaussian distribution with standard deviation equal to 10% of the exit volume. Hence,
 39 the traffic count obtained at a detector D_i is computed as

$$40 D_i(k) = q_i(k) + v_i(k) \quad (27)$$

1 Where $v_i(k) = N(0, 0.1q_i)$. D_i will be rounded if it is not a nonnegative integer.

2
3 **TABLE 1 Turning Proportions**
4

	Left	Straight	Right
Northbound	0.23	0.414	0.356
Southbound	0.29	0.352	0.29
Eastbound	0.149	0.8	0.051
Westbound	0.083	0.843	0.074

5
6 Traffic counts for 10 intervals are generated in simulation. A constrained least squares problem
7 defined by equations (5), (6) and constraints (12) - (15) is solved in every interval by optimization
8 approach CLSAS. It takes as input all the traffic counts generated from beginning to current interval. The
9 proposed method RCLS is also executed every interval. Its inputs are the latest traffic counts and the
10 estimated variables from previous interval. The initial estimate $\hat{\beta}_0$ in RCLS is obtained by random
11 positive values.

12 To evaluate the accuracy, the error between estimated turning proportions and the real ones is
13 computed at the end of each simulation run. RMSD (root-mean-square deviation) is chosen as the
14 indicator of error. The definition of RMSD is shown in equation (28). Turning proportions of all
15 movements are included in the computation of RMSD. The time for executing an algorithm on 10
16 intervals of data is collected in order to compare the computational effort. Furthermore, the frequencies of
17 applying corrections (step 2 and step 3) are used to indicate the effort RCLS pays to satisfy the constraints,
18 after its update from least squares estimator. Variables of step requiring iteration, number of iterations,
19 and step requiring truncation are computed in simulation. Step requiring iteration is the times when step 2
20 is applied in a simulation run. Number of iterations is the total count of iteration executed in a run, that is,
21 how many times step 2.3 is needed to get rid of the nonnegative values. Step requiring truncation
22 computes the times when any truncation or normalization is applied in step 3.
23

$$RMSD = \sqrt{\frac{\sum_{i=1}^N (b_{i,estimated} - b_{i,real})^2}{N}} \quad (28)$$

24
25 Table 2 shows the performance comparison between CLSAS and RCLS, and Figure 2 plots the
26 RMSD curves in experiment no. 1 to no. 4. It could be seen from Table 2 that the values of RMSD in
27 CLSAS and in RCLS are very close. Six out of ten experiments have RMSD in RCLS same as in CLSAS,
28 and the remaining have the RMDS difference only at the last digit. It is obvious from Figure 2 that, after
29 the first few steps, the RMSD curve of RCLS almost overlaps with that of CLSAS in each experiment.
30 The execution time of RCLS is only about 3% of that of CLSAS. This means that RCLS can get as good
31 estimation as CLSAS in a much smaller computational effort. It can also be found that, on average, the
32 iteration in step 2 is only executed 0.9 times out of 20 steps (two directions, and 10 intervals of data for
33 each direction). And only 3.3 iterations are applied in one experiment. Truncation or normalization is
34 conducted 2.4 times out of 20 steps. All these values indicate that the ordinary least squares estimator in
35 RCLS gets an estimate meeting the constraints most of time, and the iteration procedure in its algorithm
36 will not cost great computational effort, and therefore not influence real-time efficiency.

37 Table 3 and 4 give the estimated turning proportions at the last interval by method CLSAS and
38 RCLS, respectively. The proportion values are the averages over 10 simulation runs. The percentages in
39 the parentheses are the error percentages taken respect to the real turning proportions in Table 1. Table 3
40 and 4 are identical. This also supports the observation that RCLS gives the same estimates after a few

1 intervals. From the percentages in Table 4, it can be found that estimated values of all movements are
 2 very close to the real ones at the stationary period, which is already indicated by the small RMSD values.
 3
 4
 5

TABLE 2 Performance Comparison of CLSAS and RCLS

Experiment NO.	CLSAS		RCLS				
	RMSD	Execution time	RMSD	Execution time	Step requiring iteration	Number of iteration	Step requiring truncation
1	0.0083	0.0919	0.0083	0.0032	1	2	3
2	0.0359	0.0938	0.0358	0.0032	1	2	1
3	0.0247	0.1438	0.0247	0.0022	0	0	3
4	0.0340	0.0989	0.0338	0.0027	2	4	3
5	0.0090	0.0962	0.0091	0.0031	2	19	5
6	0.0142	0.0982	0.0142	0.0025	1	2	1
7	0.0235	0.0893	0.0235	0.0027	0	0	1
8	0.0132	0.0967	0.0134	0.0022	1	2	2
9	0.0155	0.0904	0.0154	0.0022	1	2	2
10	0.0254	0.0963	0.0254	0.0031	0	0	3
Mean	0.0203	0.0996	0.0203	0.0027	0.9	3.3	2.4

6
7

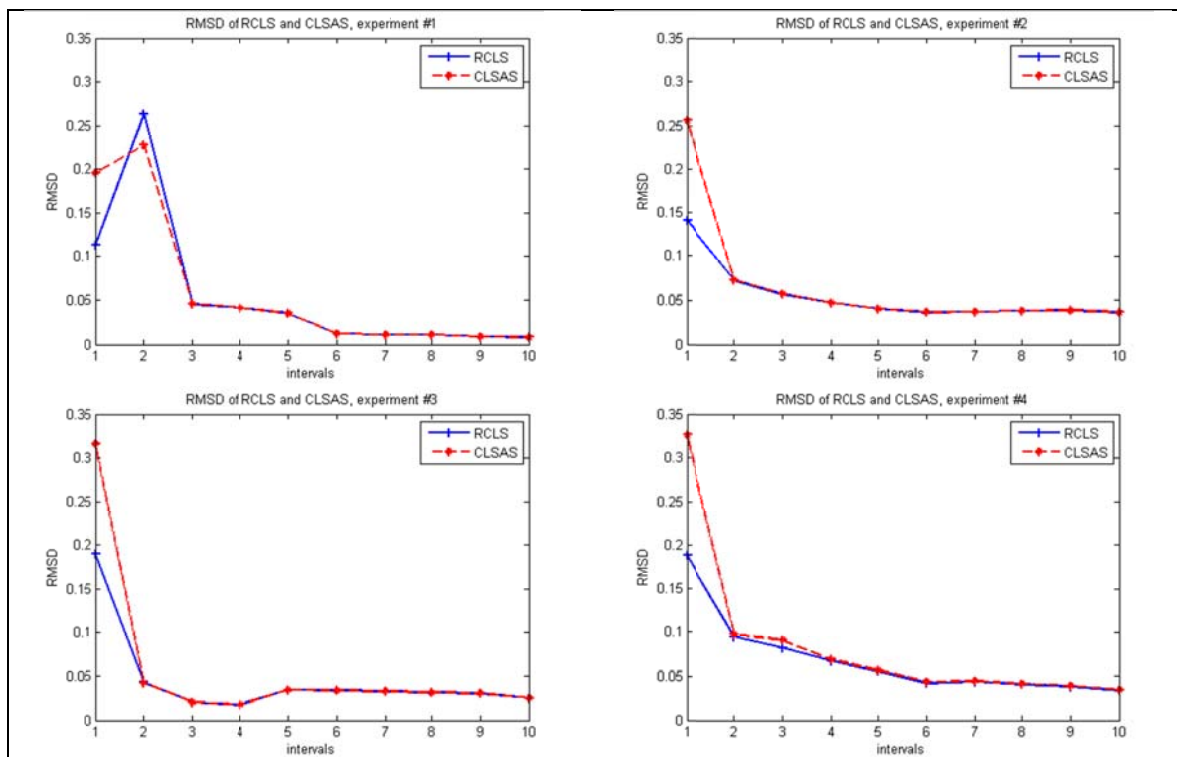


FIGURE 2 Example result of experiment #1 to #4.

8
9
10
11
12

TABLE 3 Turning Proportions Estimated by CLSASCLSAS in Scenario I

	Left	Straight	Right
Northbound	0.230 (-0.12%)	0.418 (1.07%)	0.352 (-1.17%)
Southbound	0.286 (-1.39%)	0.350 (-0.54%)	0.364 (1.66%)
Eastbound	0.146 (-1.95%)	0.799 (-0.08%)	0.055 (7.00%)
Westbound	0.083 (0.22%)	0.849 (0.75%)	0.068 (-8.77%)

TABLE 4 Turning Proportions Estimated by RCLSCLSAS in Scenario I

	Left	Straight	Right
Northbound	0.230 (-0.12%)	0.418 (1.07%)	0.352 (-1.17%)
Southbound	0.286 (-1.39%)	0.350 (-0.54%)	0.364 (1.66%)
Eastbound	0.146 (-1.95%)	0.799 (-0.08%)	0.055 (7.00%)
Westbound	0.083 (0.22%)	0.849 (0.75%)	0.068 (-8.77%)

Scenario II

In this scenario, RCLS, RCLSFR, and CLSAS will be compared in estimating time-varying turning proportions. The following simulation is conducted for the comparison. Consider the same intersection used above. The initial turning proportions are defined in Table 1 and kept for 20 intervals. These proportions are followed by the turning proportions defined in Table 5, which are kept for another 20 intervals. The process mentioned above is used to generate arrival volumes, exit volumes, and detector counts. RCLS and CLSAS are tested in the same way as above. However, adjustment has to be made to test CLSAS in this scenario. This is because, if all the traffic counts generated from beginning to current interval are input to CLSAS optimization, as what we do in the previous scenario, CLSAS will compute the aggregated turning proportions in the second period, rather than the new ratios. Only using the latest traffic counts is neither a good idea, because the estimates will be significantly affected by the randomness or noise. With such concerns, it has to be decided how many pieces of data should be fed into the optimization. From some empirical observations, it is found that using eight intervals of data gives best estimate in this test. As a result, CLSAS takes the latest 8 pieces of data in the experiments of this part. The parameters λ , ε and δ in RCLSFR also need to be determined. $\lambda=0.995$, $\varepsilon=0.0005$ and $\delta=0.0005$ are chosen because they give best estimate in tuning test.

TABLE 5 Turning Proportions in Second Period

	Left	Straight	Right
Northbound	0.32	0.544	0.136
Southbound	0.16	0.471	0.369
Eastbound	0.33	0.54	0.13
Westbound	0.2	0.5	0.3

Table 6 shows the performance of CLSAS, RCLS and RCLSFR run in this scenario, and Figure 3 plots the RMSD curves in experiment no. 1 to no. 4 in this scenario. It can be noticed that the RMSD in RCLS is about three times larger than that in CLSAS. From Figure 3, it is also observed that, even though RCLS gives a good estimation in the first period, its RMSD jumps to a very high value and drops very slowly after the turning proportions change. These values and the pattern of RMSD curve indicate that, RCLS can hardly detect the change in parameters. On the other hand, the RMSD value of RCLSFR is much smaller than that of RCLS in the second period (after turning proportions change), yet it is still larger than the RMSD value of RCLS. It is shown by Figure 3 that, the curve of RCLSFR drops faster

1 than that of CLSAS after change happened, but it fluctuates later and ends up in a larger error than
 2 CLSAS. The pattern of RMSD curves demonstrates that RCLSFR is able to detect parameter change fast,
 3 while having some fluctuation in the stationary period. The computational time of RCLSFR is about 3%
 4 of CLSAS, slightly larger than RCLS. Values for step requiring iteration, number of iteration and step
 5 requiring truncation are 1.1, 3.4, and 3.2 respectively. As indicated by these values, RCLSFR does not
 6 pay much effort to satisfy the constraints after its update from least squares estimator. Given its fast
 7 response to change and small computational effort, RCLSFR is still a competitive algorithm compare to
 8 CLSAS, even for its fluctuations in the stationary period.

9 Table 7 to 9 present the estimated turning proportions at the last interval by method CLSAS,
 10 RCLS and RCLSFR. The numbers in these tables are obtained in the same way as those in Table 3. The
 11 numbers in these tables again support the observation that CLSAS has the closest estimates, followed by
 12 RCLSFR. The estimation error of RCLS is large, due to the fact that it cannot trace the changes in
 13 variables.

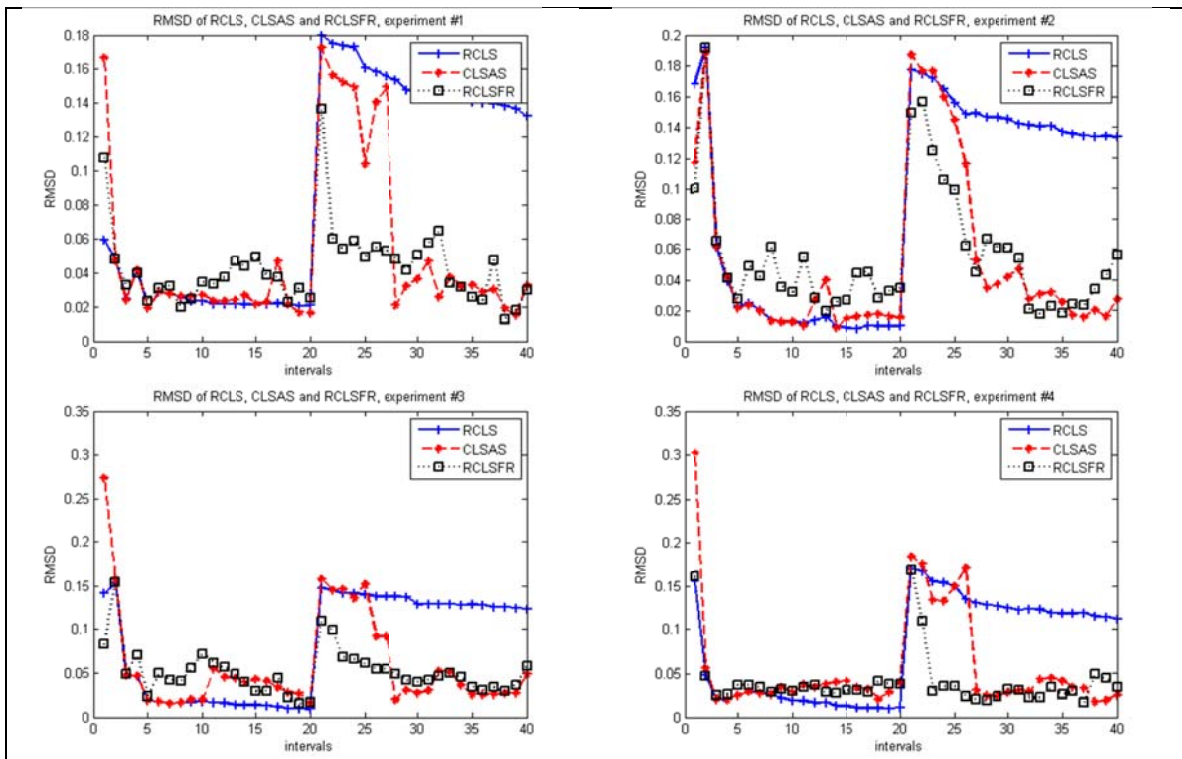
14 **TABLE 6 Performance Comparison of CLSAS, RCLS and RCLSFR**

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Experiment No.	CLSAS		RCLS					RCLSFR				
	RMSD	Execution time	RMSD	Execution time	Step requiring iteration	Number of iteration	Step requiring truncation	RMSD	Execution time	Step requiring iteration	Number of iteration	Step requiring truncation
1	0.0326	0.2991	0.1323	0.0054	0	0	0	0.0299	0.0092	0	0	3
2	0.0271	0.3793	0.1332	0.0087	2	13	3	0.0559	0.0142	5	17	6
3	0.0483	0.3051	0.1238	0.0056	2	4	2	0.0580	0.0110	1	2	3
4	0.0247	0.2895	0.1120	0.0073	1	2	2	0.0337	0.0111	2	4	6
5	0.0629	0.2984	0.1387	0.0078	0	0	2	0.0310	0.0097	2	4	4
6	0.0211	0.2918	0.1159	0.0105	0	0	1	0.0360	0.0088	0	0	1
7	0.0265	0.2903	0.1202	0.0053	0	0	1	0.0250	0.0143	0	0	3
8	0.0510	0.3025	0.1074	0.0068	1	7	1	0.0237	0.0109	1	7	2
9	0.0126	0.3018	0.0909	0.0072	1	2	1	0.0192	0.0109	0	0	2
10	0.0121	0.3247	0.0915	0.0060	0	0	1	0.0327	0.0122	0	0	2
Mean	0.0319	0.3083	0.1166	0.0071	0.7	2.8	1.4	0.0345	0.0112	1.1	3.4	3.2

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FIGURE 3 Example result of experiment #1 to #4.

TABLE 7 Turning Proportions estimated by CLSAS in Scenario II

	Left	Straight	Right
Northbound	0.320(-0.00%)	0.542(-0.33%)	0.138(1.32%)
Southbound	0.162(0.95%)	0.486(3.11%)	0.353(-4.38%)
Eastbound	0.327(-1.05%)	0.542(0.28%)	0.132(1.50%)
Westbound	0.205(2.41%)	0.505(0.95%)	0.290(-3.19%)

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TABLE 8 Turning Proportions estimated by RCLS in Scenario II

	Left	Straight	Right
Northbound	0.280(-12.58%)	0.479(-11.89%)	0.241(77.15%)
Southbound	0.218(36.21%)	0.425(-9.87%)	0.358(-3.11%)
Eastbound	0.251(-24.03%)	0.647(19.87%)	0.102(-21.55%)
Westbound	0.115(-42.71%)	0.753(50.50%)	0.133(-55.70%)

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TABLE 9 Turning Proportions estimated by RCLSFR in Scenario II

	Left	Straight	Right
Northbound	0.333(4.07%)	0.541(-0.64%)	0.126(-7.02%)
Southbound	0.151(-5.89%)	0.476(1.09%)	0.373(1.16%)
Eastbound	0.331(0.27%)	0.551(2.00%)	0.118(-9.02%)
Westbound	0.217(8.36%)	0.524(4.73%)	0.260(-13.45%)

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VI - CONCLUSION

A recursive method RCLS is proposed in this paper for estimating turning proportions of signalized intersections in real time. This method needs exit counts only, and it saves half detectors from a complete detector configuration. The estimation problem is in the form of constrained least squares, and a recursive algorithm is presented to solve it. Simulation shows that the proposed method can obtain as accurate estimation as the optimization approach CLSAS, while its computation is only 3% of CLSAS. The extended method RCLSFR, which incorporates forgetting factor and covariance resetting, is suggested for estimating time-varying turning proportions. It is able to response to change in parameters very quickly. Even though it has a bit larger error than CLSAS in the stationary period, it takes significantly less computational effort.

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Su, Horowitz, Lu

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