The Method of Averaging Using an Adaptive Notch Filter in Vibratory Gyroscopes

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***Abstract*— This paper presents an adaptive notch filter to achieve more accurate demodulation variables in the method of averaging. The main advantage of this notch filter is a very sharp attenuation gain at twice of the estimated resonance frequency, which is the main frequency component of the oscillatory noise in the demodulated variables. The position of the sharp attenuation gain can be adjusted in real time to catch the resonator's resonance frequency change. As a result, it can demodulate the control variables up to very high accuracy. Performing the method of averaging on the cleaner signals will result in much higher rotation angle estimation accuracy when the method of averaging is running in the whole angle mode. Simulation results show that with the adaptive notch filter, the oscillation in the system's quadrature and precession angle can be eliminated very well. The adaptive notch filter also effectively reduces the DC drift in the precession angle.**

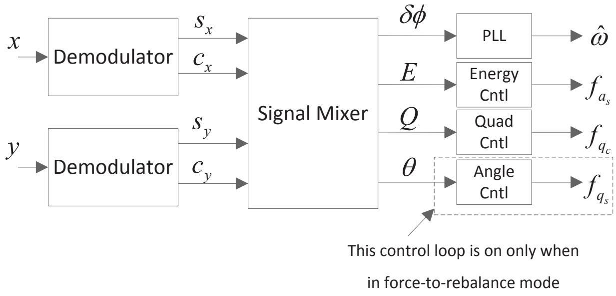

Fig. 1. Architecture of the method of averaging. This figure is adopted from [1]. $\hat{\omega}$ is the estimated resonance frequency from the phase lock loop (PLL) and other notations are consistent with [1]

I. Introduction

An effective method for controlling vibratory gyros, originally proposed by D. D. Lynch [1], is based on the method of averaging [2], and utilizes PI feedback controllers to decrease the quadrature error and maintain a constant energy oscillatory level. Fig.1 shows the architecture of the method of averaging. It consists of a demodulator, a mixer , a reference-phase loop (i.e. PLL) , an energy control loop, a quadrature control loop, and a rate control loop if the force-to-rebalance mode is enabled. The demodulator reads the sensed signals (displacements along x and y axes) and demodulates them into c_x, s_x, c_y and s_y. These demodulated variables are then synthesized into the controlled variables (i.e. phase $\delta\phi$, energy E , quadrature Q, and precession angle θ) in the mixer, and are later used in the PI feedback control loops. As a result, the gyro performance is directly affected by how well the demodulator generates the controlled variables. This will be further discussed in section II.C.

A conventional demodulator uses a Cascaded-Integrator-Comb (CIC) decimator to filter the carrier signal [3]. A CIC decimator consists of an integrator stage and a COMB stage [4]. Due to its standard architecture, CIC decimators are easy to implement using an FPGA. A CIC decimator consisting of N stages, R decimation ratio, and M samples per state has a $-20N$dB/dec amplitude attenuation slope. This is similar to that of a Nth order low pass filter, with multiple notches at $\frac{2n\pi}{RM}; n = 1, 2, \cdots$, which only depends on the structure of the CIC decimator.

However, a resonator's resonance frequency (i.e. frequency of the carrier signal) is usually varying over time. This variance is due to the changes of temperature and bias voltage that is applied on electrodes. Notches in the CIC decimator do not necessarily coincide with the resonant frequency. As a result, a significant amount of residual oscillatory noise remains in the demodulated variables. A common way of attenuating this residual oscillatory nuisance noise in the CIC decimator is increasing the number of stages N, but this will introduce a considerable phase delay in the measured external rotation angle when the gyroscope is set in whole angle mode. This requires an additional phase compensation. Meanwhile, the large N also makes the phase compensation very sensitive to the external rotation rate estimation.

In this paper, we introduce an adaptive notch filter to improve the demodulation performance. The proposed notch filter has a very sharp attenuation at twice the resonance frequency, and very little phase delay at low frequency range, which is also the input rate range. Moreover, this notch filter is very flexible, and can adjust the position of its notches accordingly with the estimated resonance frequency from PLL module. As a result, the sharp attenuation gain of the notch filter will attenuate the residual oscillatory nuisance noise in the demodulated variables at a much higher rate than what can be achieved using a CIC decimator filter. Using demodulated variables with lower residual oscillatory nuisance noise significantly reduces the angle measurement error. Simulation results are also supplied to validate the efficiency of using the proposed adaptive notch filter.

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II. DEMODULATION OF GYROSCOPE SIGNALS

A. Gyroscope Model

A vibratory gyroscope resonator is usually modeled as a two-dimensional oscillator ([1]). Its trajectory is

$$x = a\cos\theta\cos\phi - q\sin\theta\sin\phi \tag{1}$$
$$y = a\sin\theta\cos\phi + q\cos\theta\sin\phi \tag{2}$$
$$\theta = \theta_0 - k\int_0^t \Omega dt \tag{3}$$
$$\phi = \omega t + \phi_0 \tag{4}$$

where a is the oscillation amplitude along the principal axis, q is the oscillation amplitude along the quadrature axis, θ is the precession angle of the principal axis, Ω is the input rate, θ_0 is the initial angle of the principal axis, k is the gain factor of the gyroscope, ϕ is the oscillation phase, ω is the resonance frequency of the gyroscope, and ϕ_0 is the initial oscillation phase.

The resonance frequency ω of a vibratory gyroscope is usually at a much higher frequency than the precession rate Ω. This makes it possible to demodulate out the low frequency useful signals (i.e. a, q, θ) from their high frequency carrier signals (i.e. $\cos\phi$ and $\sin\phi$).

B. Structure of Demodulators

Fig.2 shows the structure of a typical amplitude modulation (AM) demodulator on channel x. Channel y is identical. In this figure, $\hat{\phi}$ is the oscillation phase estimated by the PLL; $H(z)$ denotes the transfer function of the low pass filter that is used to filter out the carrier signal.

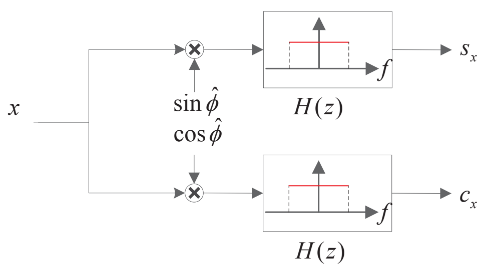

Fig. 2. Structure of a typical AM demodulator on channel x. $\hat{\phi}$ is the oscillation phase estimated by PLL

The demodulated variables are therefore:

$$s_x = H(z)[x\sin(\hat{\phi})] \tag{5}$$
$$c_x = H(z)[x\cos(\hat{\phi})] \tag{6}$$
$$s_y = H(z)[y\sin(\hat{\phi})] \tag{7}$$
$$c_y = H(z)[y\cos(\hat{\phi})] \tag{8}$$

Substituting the trajectory of x and y (i.e. Eq.(1) and Eq.(2)) into the above equations, we break the multiplied signal in each channel into two parts: useful signal for extraction (usually at very low frequency) and noise signal for attenuation (usually at very high frequency).

$$s_x = H(z)[\frac{a\cos(\theta)\sin(\delta\phi) - q\sin(\theta)\cos(\delta\phi)}{2} + A_{xs}\sin(2\hat{\phi} + \phi_{xs})] \tag{9}$$
$$c_x = H(z)[\frac{a\cos(\theta)\cos(\delta\phi) + q\sin(\theta)\sin(\delta\phi)}{2} + A_{xc}\sin(2\hat{\phi} + \phi_{xc})] \tag{10}$$
$$s_y = H(z)[\frac{a\sin(\theta)\sin(\delta\phi) + q\cos(\theta)\cos(\delta\phi)}{2} + A_{ys}\sin(2\hat{\phi} + \phi_{ys})] \tag{11}$$
$$c_y = H(z)[\frac{a\sin(\theta)\cos(\delta\phi) - q\cos(\theta)\sin(\delta\phi)}{2} + A_{yc}\sin(2\hat{\phi} + \phi_{yc})] \tag{12}$$

An ideal low pass filter $H(z)$ should have a gain of 1 with zero phase delay at low frequency range, and a gain of 0 at high frequency range. This ideal low pass filter will extract the useful signal exactly. However, a practical low pass filter usually has phase delay at low frequency, and causes the estimated precession angle θ to be delayed. This phase delay should be compensated later in angle estimation in the whole angle mode. In addition, a practical low pass filter usually has non-zero gain at high frequencies, causing the demodulated variables to have some residual high frequency oscillatory noise. The frequency of this oscillatory noise is around $2\hat{\omega}$ (twice of the resonance frequency estimated from PLL), as shown in Eq.(9-12).

C. Effect of oscillatory noise on gyroscope performance using the method of averaging

The oscillatory noise in the demodulated variables c_x, s_x, c_y and s_y will propagate through the mixer and affect the energy estimation E, quadrature estimation Q and phase error $\delta\phi$ by Eq.(31) in [2]. As a result, the estimated energy, quadrature, and phase error will also oscillate at $2\hat{\omega}$. Then by Eq.(42) in [2], the oscillatory energy, quadrature, and phase error will cause the energy control action f_{a_s}, quadrature control action f_{q_c} and phase estimation $\hat{\phi}$ in PLL to oscillate at $2\hat{\omega}$ as well.

When the whole angle mode is enabled in the method of averaging, the principal oscillation axis is free to precess. Its governing equation is given by the θ equation in Eq.(42) of [2], where f_{q_s} is zero. Since both f_{q_c} and $\delta\phi$ are oscillating at $2\hat{\omega}$ and $\delta\phi$ is small, the term $f_{q_c}\sin\delta\phi \approx f_{q_c}\delta\phi$, can be breaken into a DC term and an oscillatory part at $4\hat{\omega}$. Since there is no control action to regulate the precession angle θ, this DC term will act directly on θ's dynamics and cause it to drift.

To attenuate the precession drift, a proper demodulator has to be designed to attenuate the residual oscillatory noise in the demodulated variables.

III. Adaptive Notch Filter Design

A. A Basic Notch Filter

A basic digital notch filter has the transfer function

$$H(z) = \frac{z^2 + 2\alpha\cos(\omega_0)z + \alpha^2}{z^2 + 2\beta\cos(\omega_0)z + \beta^2} \quad (13)$$

where z is the Z-transform variable, and α, β and ω_0 are the design parameters.

To attenuate the noise at frequency ω_0, we usually set $\alpha = 1$, which results in $H(z)\sin(\omega_0 k) = 0$ at steady state. Then by setting $1 \geq \beta \geq 0$, the notch filter has a V-shape notch at the frequency ω_0. Fig.3 shows how the depth and width of the V-shape notch changes as a function of β in the notch filter. As β increases the notch becomes wider and wider. When $\beta = 1$, $H(z) = 1$ and the notch is flatted out.

As shown in Fig.3, by properly choosing α and β, the notch filter has very large attenuation gain at the specified frequency ω_0 , and a very small phase delay at low frequency range. This property makes the notch filter an excellent choice for demodulation in the gyroscope system.

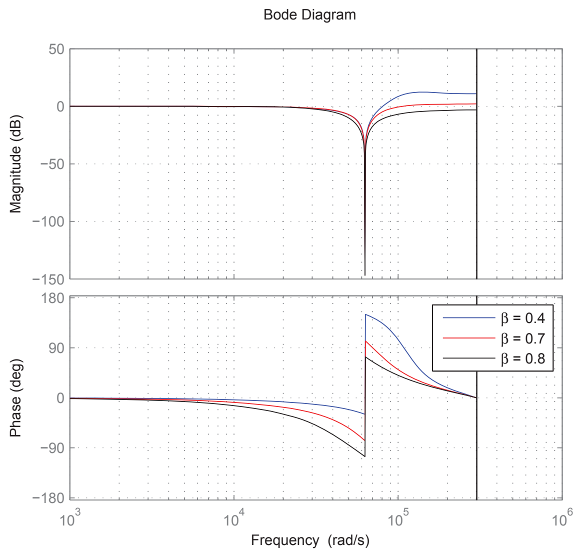

Fig. 3. Bode diagram of a notch filter with $\alpha = 1.0, \beta = 0.7$ and $\omega_0 = 0.6545$, sampling frequency is $F_s = 96kHz$

Fig.4 shows the comparison of frequency response between a CIC filter and a notch filter. First we can see that compared with the CIC filter, the notch filter has almost no delay at low frequency signal, thus causing little phase delay on the demodulated true signals. Also the notch filter has much larger attenuation gain than the CIC filter at the specified frequency, thus having much better SNR.

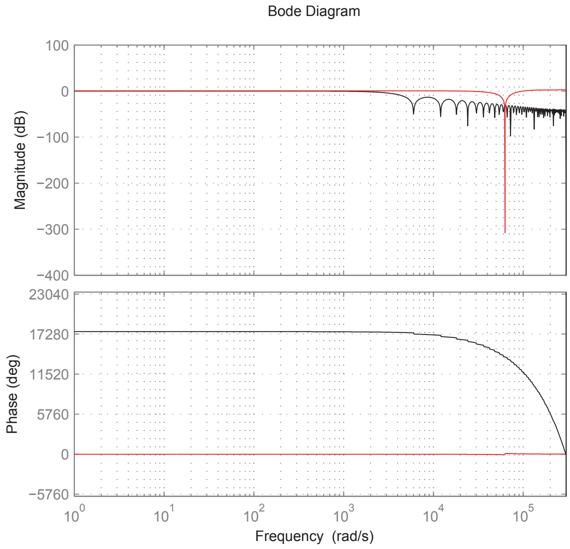

Fig. 4. A comparison of frequency response between CIC filer and notch filter. In this CIC filter, number of stage $N = 1$, decimation ratio $R = 100$, delay steps in comb state $M = 1$. In the Notch filter, $\alpha = 1.0, \beta = 0.7, \omega_0 = 0.6545$. In both of them, the sampling frequency $F_s = 96kHz$

B. Incorporating An Adaptive Notch Filter into the Method of Averaging

The notch filter can dramatically attenuate signals at its notch frequency ω_0, and the demodulated variables c_x, s_x, c_y and s_y have residual oscillatory carrier signals at $2\hat{\omega}$ shown in Eq.(9-12). If the notch frequency ω_0 is specified to be $2\hat{\omega}$ in the adaptive notch filter, then we should be able to attenuate the residual oscillatory carrier signals dramatically, thus getting very clean demodulated variables. The resulting notch filter is therefore

$$H(z) = \frac{z^2 + 2\alpha\cos(2\hat{\omega})z + \alpha^2}{z^2 + 2\beta\cos(2\hat{\omega})z + \beta^2} \quad (14)$$

where $\hat{\omega}$ is drawn from the PLL in real time. This means the adaptive notch filter adjusts its coefficients by $\cos(2\hat{\omega})$ at each time $\hat{\omega}$ is updated from the PLL.

In practice, β has to be chosen carefully. Ideally, we want the notch to be wide enough to tolerate the resonance frequency estimation error $(\hat{\omega} - \omega)$ caused by the transient or steady error from the PLL. On the other hand, we want the notch to be as deep as possible to attenuate the residual oscillatory carrier signals. However, as discussed in last subsection, a large β has a wider but shallower notch, and a small β has deeper but narrower notch. While the transient response of the PLL is determined by the bandwidth of the PLL, to make sure the estimated resonance frequency converges to the real resonance frequency, β should be chosen carefully, according to the bandwidth of the PLL.

C. Adaptive Notch Filter Implementation

The proposed adaptive notch filter is a standard second order IIR filter, and can be easily implemented in a FPGA or DSP. Since the PLL in the method of averaging is running at a very high sampling rate, it requires that the adaptive notch filter compute its coefficients $\cos(2\hat{\omega})$ very efficiently. If the adaptive notch filter is implemented in a DSP using C/C++, computation of the cosine function can be very in efficient if using the standard C math library. Fortunately, computational improvements can be made by using the DSP specific math library. An alternative method is using a cosine look-up table. The look-up table is built by filling its entries with the cosine values of points that are equally distributed between $[0, 2\pi]$, and the evaluate the cosine function by a first order interpolation method in the lookup table.

IV. Simulation

A computer simulation study was conducted using a gyroscope model to validate the proposed adaptive notch filter. In the simulation the gyroscope resonance frequency used is $\omega = 10kHz$. The gain factor is set to be 1 and there is no mismatch considered in the model. The external input rate is a constant at $1Hz$. The low pass filter is a first order low pass filter with bandwidth of $100Hz$. The numerical values for the Notch filter used in the simulation are $\alpha = 1.0$, $\beta = 0.7$, and $\omega_0 = 2\hat{\omega}$, where $\hat{\omega}$ is the estimated resonance frequency drawn from PLL in real time. The the sampling frequency is $F_s = 96kHz$. The gyro runs in the whole angle mode. Fig. 5 shows the simulated quadrature and Fig. 6 shows the simulated angle estimation error. As can be seen, the residual carrier signal is significant in the lowpass case. However, the adaptive notch filter eliminates the residual oscillatory error. The residual carrier signal in quadrature introduces ramp bias in the angle measurement. Fig. 6 compares the estimated error in the low pass filter and notch filter respectively. The results depict that the adaptive notch filter eliminates the Ramp bias and improves the angle estimation drastically.

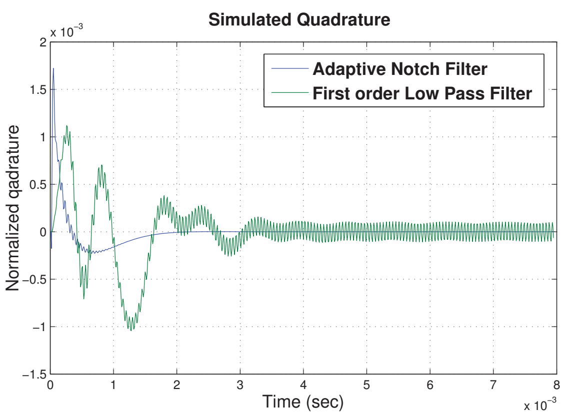

Fig. 5. Simulated quadrature of the lowpass and adaptive notch filter

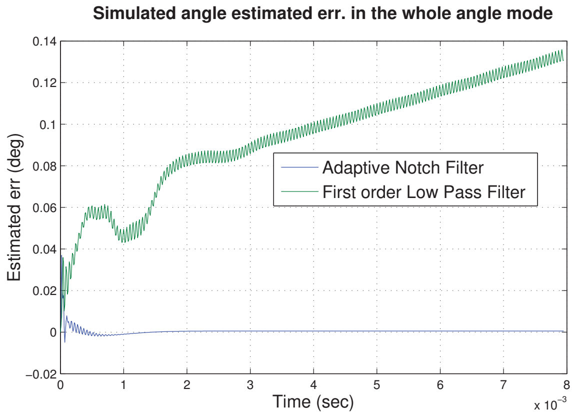

Fig. 6. Simulated angle estimated error of the lowpass and adaptive notch filter

V. Conclusion

In this paper, we introduced an adaptive notch filter. The notch filter has a very large attenuation gain at the notch frequency and little phase delay at low frequency. The notch frequency is adapting in real-time based on the estimation of the resonator's resonance frequency. Utilizing the proposed notch filter in the method of averaging can drastically attenuate the oscillatory nuisance noise in the demodulated variables. A simulation study using an ideal gyroscope model was conducted to verify the performance of the proposed method. Simulation results demonstrate that using the adaptive notch filter improves the accuracy of rotation angle measurement.

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