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A probabilistic approach to robust controller design for a servo system with irregular sampling

Ehsan Keikha[1], Behrooz Shahsavari[2], Abdullah Al-Mamun [1] and Roberto Horowitz[2]

Abstract— **This paper deals with the problem of irregular sampling rate in the discrete time track following control of read/write head in hard disk drives (HDD). Irregular sampling is gradually becoming an issue not to be ignored in modern HDDs. Current technological trend in the industry suggests an increasing need for assembling disks prewritten with servo patterns into the drive. As a result, misalignment between center of servo tracks and center of rotation is an unavoidable consequence. In this paper, a non-sequential probabilistic algorithm based on scenario is utilized to handle uncertainties with sampling rate in HDD. Effectiveness of the designed controller is underscored by simulation results.**

I. INTRODUCTION

Sampled-data systems with a known sampling interval have been studied extensively over the past decades and the systems can be analyzed in the discrete time . When the sampling interval is unknown, the situation is more complicated. Time-varying sampling is applied to many practical systems. For instance networked control systems which have emerged as a hot research area provide a convincing background, where conflicts may exist and inevitably degrade control performance due to external traffic and the limitation of the network resource. Under such conditions, it may improve the system performance that the sampling period varies with the network condition.

Conventionally the servo system of the Hard Disk Drive (HDD) is modeled as an LTI system [1]. However, time invariance of the HDD servo system is not a valid assumption as the sampling rate in HDD may vary due to various reasons. Irregular sampling will be a more frequent problem when disks are prewritten with servo patterns as in case of discrete-track HDD or bit patterned media HDD [2]. In these cases, center of the rotation of disk does not coincide with the center of the servo tracks. With such misalignement between center of disk and center of rotation, there can be considerable variations in the sampling rate of the position error signal (PES) [3]. Irregular sampling may also be the outcome of other problems, e.g., media defects in servo sectors, fast disk motion causing incorrect detection of servo address mark (SAM) etc contributing to false PES demodulation. The feedback PES is unavailable in those servo sectors, resulting in an irregular sampling [4]. In general, there are two possible ways to update the control action for a system with irregular sampling. In the first scheme after a measurement is obtained, the control action is updated as quickly as possible making the control update rate to be irregular as well. In the other scheme, the control action is updated at a fixed, regular rate. In the case of the HDD second scheme is more attractive since the output of the controller is usually passed through one or more notch filters, and the notch filters can be designed independent of the sampling rate variation if this scheme is used.

In the recent past, designing a track following controller using a robust approach based on Linear Matrix Inequality (LMI) has received considerable attention from researchers and industries. These methods show promising results to improve the overall track misregistration (TMR) performance. However if the number of parametric uncertainties increases these methods become computationally complex and conservative [5]. In the case of a linear system with irregular sampling rate, most of the parameters in the state space realization of the system become uncertain and it may not possible to find an analytical solution with the current LMI solver available. Moreover even if we limit the size of uncertainty set the solution become too conservative and not tally to the initial problem.

To overcome these limitations of robust control design, a new framework known as probabilistic approach was introduced [6]. This approach is analogous to worst case design and it employs sequential and nonsequential algorithms. Among the sequential algorithms gradient [7], ellipsoid [8] and cutting plane [9] iterations can be mentioned which can handle convex feasibility problems. The scenario approach of Calafiore and Campi [10] is a non-sequential technique which can solve in a probabilistic sense any general uncertain convex optimization problems.

The main contribution of this paper is to propose a robust probabilistic control design methodology when the sampling rate is irregular but the control rate is regular. This approach is based on a scenario probabilistic design which is a non-sequential randomized technique for achieving probabilistic robustness in design. Then we use this method to address the problem of the track following controller design in a HDD with irregular sampling. Final simulation results show that the designed controller can achieve required performance while maintaining good robustness.

II. MODEL DISCRETIZATION

In this section, we will describe the discretization of the system with irregular sampling and regular control action

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[1]Ehsan Keikha and Abdullah Al-Mamun are with Electrical & Computer Engineering Department, National University of Singapore (e-mail: Ehsan.keikha@nus.edu.sg , e-mail: Eleaam@nus.edu.sg).

[2]Behrooz Shahsavari and Roberto Horowitz Mechanical Engineering Department, UC Berkeley, CA 94720 USA (e-mails: behrooz@berkeley.edu, horowitz@berkeley.edu)

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from a continuous-time model. Throughout this section, the continuous-time state space model of the system is given by

$$\dot{x}_c(t) = A_c x_c(t) + B_c u_c(t) \quad (1)$$
$$y_c(t) = C_c x_c(t) + D_c u_c(t) \ . \quad (2)$$

The signals $x_c(t)$, $u_c(t)$, and $y_c(t)$ respectively have dimension n_x, n_u, and n_y. The computational delay associated with the controller will be denoted as δ. We also define for $T > 0$

$$A_d(T) := e^{A_c T}, \qquad B_d(T) := \left(\int_0^T e^{A_c \tau} d\tau \right) B_c \ .$$

Note in particular that these matrices are the discrete-time state space matrices that result when discretizing the continuous-time state dynamics using a zero-order hold with period T. Throughout this section, we will use a zero-order hold approach to discretizing the model. We consider the case when the measurements are sampled at an irregular (but periodic) rate and the control action is updated at a regular rate, i.e. we choose $t = 0$ so that the control is updated at time instances

$$t_{u,k} = kT, \ k \in \mathbb{Z} \, ; .$$

Due to computational delay, the control signal is not updated at the same time as measurements are obtained. We will assume for simplicity that $\delta < T$, where T is the sample period. Since the control updates are scheduled in time rather than the control being updated as quickly as possible after receiving a measurement, we treat the computational delay as the constraint that u_k can only depend on any measurements that arrive in the time interval $(-\infty, \ t_{u,k} - \delta]$. Equivalently, the value of u_k will be updated at time step $k+1$ by measurements that arrive in the time interval

$$S_k := (t_{u,k} - \delta, \ t_{u,k+1} - \delta] \ .$$

We now define x_k and u_k as given in (3).

$$x_k := x_c(t_{u,k}); \quad u_k \quad := u_c(t_{u,k}); \quad y_k := y_c(t_{y,k}) \quad (3)$$

The zero-order hold assumption yields that

$$u(t) = u_k, \qquad t \in [t_{u,k}, \ t_{u,k+1})$$

which means that (1) discretizes as

$$x_{k+1} = A_d(T) x_k + B_d(T) u_k \ . \quad (4)$$

To find the discrete-time representation of the measurements, we first note that

$$t_{y,k} = t_{u,k} + (T - \delta)$$

which means that

$$y_k = C_c x_c(t_{u,k} + (T-\delta)) + D_c u_c(t_{u,k} + (T - \delta)) \ .$$

Since the control is updated regularly in time, we discretize (1) as

$$x_{k+1} = A_d(T) x_k + B_d(T) u_k \ .$$

These dynamics are LTI and do not depend on the measurement characteristics of the system. We now find the representation of a measurement at a time instant $\bar{t} \in S_k$. There are two cases to consider. The first case corresponds to $\bar{t} \geq t_{u,k}$. In this case, we obtain

$$\begin{aligned} y(\bar{t}) &= C_c x_c(\bar{t}) + D_c u_c(\bar{t}) \\ &= [C_c A_d(\bar{t} - t_{u,k})] x_k + [C_c B_d(\bar{t} - t_{u,k}) + D_c] u_k \end{aligned}$$

The second case corresponds to $\bar{t} < t_{u,k}$. In this case, we note that

$$\begin{aligned} x_c(\bar{t}) &= A_d(\bar{t} - t_{u,k-1}) x_{k-1} + B_d(\bar{t} - t_{u,k-1}) u_{k-1} \\ &= A_d(\bar{t} - t_{u,k-1}) A_d^{-1}(T) [x_k - B_d(T) u_{k-1}] \\ &\quad + B_d(\bar{t} - t_{u,k-1}) u_{k-1} \ . \end{aligned}$$

For notational convenience, we define

$$\begin{aligned} \bar{A} &:= A_d(\bar{t} - t_{u,k-1}) A_d^{-1}(T) \\ \bar{B} &:= B_d(\bar{t} - t_{u,k-1}) - A_d(\bar{t} - t_{u,k-1}) A_d^{-1}(T) B_d(T) \end{aligned}$$

so that the previous expression can be written as

$$x_c(\bar{t}) = \bar{A} x_k + \bar{B} u_{k-1} \ .$$

This yields

$$\begin{aligned} y(\bar{t}) &= C_c x_c(\bar{t}) + D_c u_c(\bar{t}) \\ &= C_c \bar{A} x_k + C_c \bar{B} u_{k-1} + D_c u_k \ . \end{aligned}$$

Since this expression depends on u_{k-1}, we need to augment the state vector, i.e. we write the state dynamics of the discrete-time system as

$$\begin{bmatrix} x_{k+1} \\ u_k \end{bmatrix} = \begin{bmatrix} A_d(T) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} B_d(T) \\ I \end{bmatrix} u_k \ . \quad (5)$$

Again, these dynamics are LTI and do not depend on the measurement characteristics of the system. The output at time instant $\bar{t} \in S_k$ corresponds to

$$y(\bar{t}) = \bar{C} \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} + \bar{D} u_k \quad (6)$$

where

$$\bar{C} = \begin{cases} \begin{bmatrix} C_c \bar{A} & C_c \bar{B} \end{bmatrix}, & \bar{t} \in (t_{u,k} - \delta, \ t_{u,k}) \\ \begin{bmatrix} C_c A_d(\bar{t} - t_{u,k}) & 0 \end{bmatrix}, & \bar{t} \in [t_{u,k}, \ t_{u,k+1} - \delta] \end{cases} \quad (7)$$

$$\bar{D} = \begin{cases} D_c, & \bar{t} \in (t_{u,k} - \delta, \ t_{u,k}) \\ C_c B_d(\bar{t} - t_{u,k}) + D_c, & \bar{t} \in [t_{u,k}, \ t_{u,k+1} - \delta] \end{cases} \quad (8)$$

Under the assumption that the sampling of the system is periodic, (5)–(8) define an LPTV discrete-time state space model without loss of generality we can assume that the period of the system is large enough to model it as linear time varying system . This model describes the continuous-time model (1)–(2) under a zero-order hold on the input with sampling and actuation conditions described at the beginning of this section.

III. Controller Design

The model of the system which described in the previous section resulted in a periodically time varying (LPTV) model of the HDD with a large period which is equal to the number of servo sectors in one revolution. As a result if we want to design an LPTV controller for the system the total number of controllers which is required to store the memory of servo system become large. In a physical HDD, there is a limited amount of memory available to store the controller parameters and it is almost impossible to reserve so much memory for storing these parameters in real HDDs so the LPTV controller designed has some limitation in the implementation. In this section, we present an LTI H_2 control design for a HDD model with irregular sampling and regular control action. We are assuming that the sampling time is uncertain parameter which in norm bounded. However by assuming uncertainty in the sampling time the problem falls into the category of the so-called "intractable" problems, which are practically unsolvable if the number of variables becomes sufficiently large (which is the case in the HDD controller design problem). To address this issue we use the idea of probabilistic framework. In probabilities robust control design which randomize algorithms are employed to handle uncertainty and convex optimization to compute the design parameters. We follow a two step design in order to obtain the final controller. In the first step, a classical H_2 controller without considering any uncertainty is formulated. Next, we employ a randomized algorithm which called scenario approach [11] to overcome the uncertainty presented in the problem. Our control objective throughout this paper is to maximize the stability margins while keeping the position error signal, PES, as small as possible, in order to achieve high areal densities and low-readout error rates. The architecture we use for control design when the sampling rate is regular or irregular in time is shown in Fig. 1. The signals r; w and n are independent white noises with unit variance that respectively model the effect of the non-repeatable runout (NRRO), windage, and measurement noise. NRRO is the random lateral movement of the disk caused by the mechanical contacts in the bearing motor, and windage is the off track motion in the head caused by the turbulent nature of the air between the disk and the actuator[1].
The signals p, y and u are respectively the performance, measurement and control signals. Note that the performance and measurement signals are both equivalent to the PES. The blocks K, NRRO, σ_n and σ_w are respectively the controller, the NRRO model, the standard deviation of measurement noise, and the standard deviation of the windage.

A. Nominal H_2 design

Consider the augmented open-loop plant shown in Fig. 2 with the state space representation.

$$P: \quad \begin{bmatrix} z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_w & B_u \\ \hline C_{ew} & D_{ew} & D_{eu} \\ C_y & D_{yw} & 0 \end{array}\right] \begin{bmatrix} w \\ u \end{bmatrix} \tag{9}$$

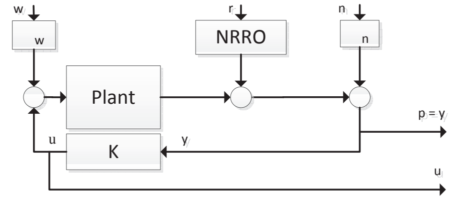

Fig. 1. Closed loop system for performance calculation

The goal is to design a robust dynamic output feedback controller in the form

$$u = K(s)y = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & 0 \end{array}\right] y \tag{10}$$

which minimizes the worst case H_2 norm from disturbance w to the output$z$. In other words, we are interested in solving the following optimization problem:

$$K = \arg \underset{K \in \mathcal{K}}{\text{minimize}} \, \|T_{zw}\|_2 \tag{11}$$

where T_{zw} is the closed loop transfer function from disturbance w to the output z. The optimization problem (11) can be reformulated in terms of linear matrix inequality (LMI) [12] as follows:

$$\underset{\mathcal{W},\mathcal{P},\mathcal{K},\gamma}{\text{minimize}} \ \gamma \ \text{ subject to } \ \Omega\left(\mathcal{W},\mathcal{P},\mathcal{K},\gamma\right) \succ 0 \tag{12}$$

where

$$\Omega\left(\mathcal{W},\mathcal{P},\mathcal{K},\gamma\right) = \left(\gamma^2 - trace(\mathcal{W})\right) \oplus \begin{bmatrix} \mathcal{W} & C_{cl}\mathcal{P} \\ \star & \mathcal{P} \end{bmatrix} \oplus \begin{bmatrix} \mathcal{P} & A_{cl}\mathcal{P} & B_{cl} \\ \star & \mathcal{P} & 0 \\ \star & \star & I \end{bmatrix} \succ 0 \tag{13}$$

where $\mathcal{P} = \mathcal{P}^T$, $\mathcal{W} = \mathcal{W}^T$ and $\mathcal{K}$ are design variables and A_{cl}, B_{cl} and C_{cl} are uncertain closed loop matrixes. Introducing the uncertain in the sampling time in the formulated LMI Equation. (13) will make the problem extremely difficult to solve. In special case where the uncertain parameter enters affinely into the LMI, we can solve the the problem for all vertices of the uncertainty set and the solution is guaranteed to be feasible for the entire set of the uncertainty. However, in the case where the uncertainty enters into the optimization problem in a non-affine manner, we cannot use the same strategy. In some special cases we can use relaxation techniques to solve the problem which is associated with conservatism and the rate of conservatism introduced by the relaxation is in general unknown. In a probabilistic framework, by accepting a very small risk that the constraint equation 13 is being violated, we can solve the uncertain problem easily. In the following subsection, we briefly review a successful probabilistic which recently has been introduced.

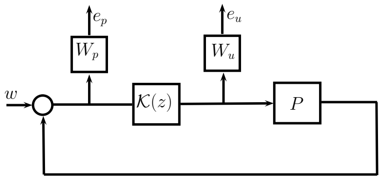

Fig. 2. Augmented Plant

TABLE I
IDENTIFIED MODAL PARAMETERS

n	ω_i (rad/sec)	ζ_i	A_i
1	$2\pi \times 22$	0.08	7.6e8
2	$2\pi \times 6063$	0.02	-1.5e9
3	$2\pi \times 9358$	0.06	-1e9
4	$2\pi \times 15693$	0.02	-4.5e9

B. The Scenario Approach

The main idea behind the probabilistic framework is to use randomization to handle uncertainty and convex optimization to compute the design parameters. In this approach, we assume that uncertainty is a random variable q with probability density function (pdf) $f_q(\boldsymbol{q})$ and support $B_{\mathbb{D}}(\rho)$ where $B_{\mathbb{D}}(\rho)$ is uncertainty ball with radius ρ. Lets define the set of LMIs obtained after change of variable and non-linear transformation as

$$f(\theta, q) \prec 0 \tag{14}$$

The aim is to find design parameter θ which satisfies the LMI constraint (14) in a probabilistic fashion. In other words, we want to find θ such that:

$$Pr\left\{f(\theta, q) \prec 0\right\} \geqslant 1 - \varepsilon$$

It turns out that this problem is extremely difficult to solve *exactly*; since it requires the computation of multi-dimensional integrals. However, we can *estimate* the probability using randomization. Because of introducing randomization, there will be another probabilistic level (q) which indicates the probability bound that randomized algorithm may come up with an erroneous result. Therefore, the full objective is to find θ such that:

$$Pr_{q^{1,\dots,N}}\left\{Pr\left\{f(\theta, q^{(i)}) \prec 0\right\} \geqslant 1 - \varepsilon\right\} \geqslant 1 - \delta \tag{15}$$

In general, there are different methodologies to handle this problem [13], [7], [10]. In this paper, we use the non-sequential approximation method based on scenario approach [10]. In this approach, a presanctified number of randomly selected independent scenarios are chosen and optimization is solved for this finite number of samples instead of the infinite one. In order to solve the problem (15), a particular function $\tau(\theta, q)$, related to the objective function (14), is introduced. Essentially, the function $\tau(\theta, q)$ measures the level of violation of performance function and is called *performance violation function*. The following properties hold for performance Here, we briefly study the so called scenario approach also known as random convex programs. Suppose that the goal is to solve the following uncertain convex problem

$$\begin{aligned} &\underset{\theta\in\Theta}{\text{minimize}} \quad c^T\theta \\ &\text{subject to } F(\theta, q) \leq 0 \text{ for all } q \in \mathbb{Q} \end{aligned} \tag{16}$$

where $\theta \in \Theta \subset \mathbb{R}^{n_\theta}$ is the vector of optimization variables and $q \in \mathbb{Q} \subset \mathbb{R}^{\ell}$ is the vector of problem data which is uncertain. We assume $f(\theta, q)$ to be convex for any fixed value of q. In order to solve (16) using the scenario approach, we need to extract a multi-sample $\mathbf{q} = \{q^{(1)} \dots q^{(N)}\}$ of cardinality N from the uncertainty set and form the following random convex program:

$$\begin{aligned} \widehat{\theta}_N = \arg\underset{\theta\in\Theta}{\text{minimize}} \quad & c^T\theta \\ \text{subject to} \quad & f(\theta, q^{(i)}) \leq 0, \quad i = 1, \dots, N. \end{aligned} \tag{17}$$

The following definition is used in order to define the probabilistic behavior of the solution obtained from (17).

Definition 1 (Probability of Violation)**.** *The probability of violation of θ for the function $f(\theta, q$ is defined as:*

$$E_f(\theta) \doteq \Pr\left\{q \in \mathbb{Q} \;:\; f(\theta, q) > 0\right\}. \tag{18}$$

The following lemma which is proved in ([14]) defines the explicit sample bound N which bounds the probability of violation of $\widehat{\theta}_N$.

Lemma 1. *Suppose that the convexity assumption holds and let $\delta, \varepsilon \in (0, 1)$ and N satisfy the following inequality:*

$$\sum_{i=0}^{n_\theta - 1} \binom{N}{i} \varepsilon^i (1 - \varepsilon)^{N-i} \leq \delta. \tag{19}$$

Then if the optimization problem (17) is feasible with probability no smaller than $1 - \delta$, its optimal solution $\widehat{\theta}_N$ satisfies $E_f(\widehat{\theta}_N) \leq \varepsilon$.

There are a number of results in the literature for deriving the explicit sample size N which guarantees (19). The least conservative one which is reported in [10] is

$$N \geq \inf_{a>1} \frac{1}{\varepsilon}\left(\frac{a}{a-1}\right)\left(\ln\frac{1}{\delta} + (n_\theta - 1)\ln a\right). \tag{20}$$

We used the studied scenario approach in order to solve the LMI (13).

IV. Simulation study

This section presents numerical simulation results to show the effectiveness of the proposed randomized algorithm to design a track-following controller for in a hard disk drive with irregular sampling. Our simulation is based on a model for a 3.5-inch hard disk drive.

Fig. 3 shows the measured frequency response as well as the estimated models of VCM. The experimental data show

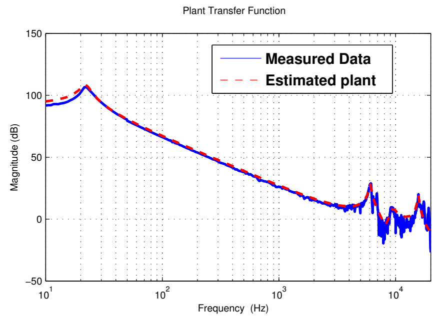

Fig. 3. Measured and the identified frequency response for VCM actuator

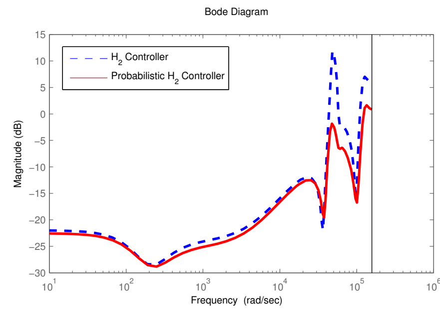

Fig. 4. Comparison between the nominal H_2 controller and the designed probabilistic controller

several distinct resonance modes. As a result, it is appropriate to describe the transfer function between each input-output pair as a summation of n modes, as follows. The transfer function of VCM is considered to be in the form of:

$$G_{VCM} = \sum_{i=1}^{4} \frac{A_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \tag{21}$$

where ζ_i, ω_i and A_i are damping ratio, natural frequency and modal constant respectively. Table I shows the estimated nominal plant parameters.

The next step is to discretize the plant based on the method proposed in the section II. We have assumed that the sampling time is irregular so in the plant discretization we have assumed that the sampling time uncertainty is expressed in the multiplicative form as $t_{u,k} = \bar{t}_{u,k}(1 + 0.16\eta_1)$ where parameters $\bar{t}_{u,k}$, denotes the nominal values and η_1 is perturbations which are norm bounded. We assumed uniform probability density, due to its worst case nature while sampling the uncertainty space.

The block diagram of the augmented open loop system is shown in Fig.2. P Denotes the nominal plant, W_u, and W_p are the weighting functions and $\mathcal{K}(s)$ is the dynamic output feedback controller to be designed. Since the solution of the optimization problem strongly depends on the design parameters (e.g. weighting functions) we use the following 2-step algorithm to find the desired controller. Initially the weighting functions need to be defined for the nominal LTI plant and based on the Equation 12 the corresponding nominal controller is designed. The control weighting functions W_u is chosen to be 0.2 and the performance weighting functions W_p is chosen to be in the form of$W_p = \left(\frac{s/\overline{S}+w_c}{s+w_c\underline{S}}\right)^k$ for which w_c can determine the crossover frequency (desired bandwidth), $\overline{S}$ determines the desired bound on the sensitivity peak, $\underline{S}$ determines the minimum level of the closed loop sensitivity function in low frequencies and finally, k determines the slope of the closed loop sensitivity function.

After the nominal controller designed the the results are given to the randomized algorithm Equation 17 as an initial value. The output of the algorithm is the controller parameters that make the closed-loop plant robustly stable (in the probabilistic sense). The bode plot of the nominal as

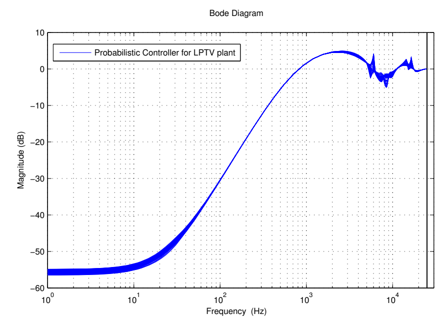

Fig. 5. Closed loop sensitivity plot for 500 random realizations for the system with probabilistic H_2 controller

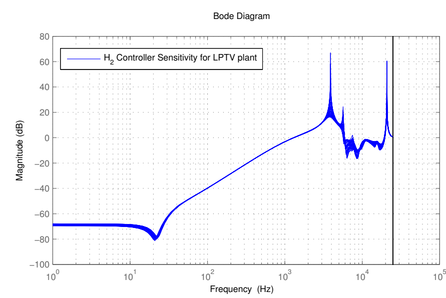

Fig. 6. Closed loop sensitivity plot for 500 random realizations for the system with nominal H_2 controller

well as designed robust controllers, which is of order 10, are shown in Fig 4 for a better comparison.

As mentioned earlier, the objective is to design a dynamic output feedback controller $\mathcal{K}(s)$ which minimizes the worst case (over the uncertainty set) $\mathcal{H}_2$ norm of the transfer function of exogenous signal w to outputs e_p and e_u. In order to demonstrate the robustness of the designed controller we extracted 500 random samples from the uncertainty set and formed the error rejection (sensitivity) transfer function for each random realizations. Fig.5 shows the obtained sensitivity transfer function for 500 uncertain scenarios. As it is clear, the sensitivity transfer function does not degrade over the uncertainty set which proves the robustness of the designed controller. Fig. 6 shows that the controller for the nominal plant tend to instability for some uncertain realizations. Since the problem is of regulation type, the

Design Approach	*TMR* (nm)		*RMS*(u_V) *(mV)*		$\|u_V\|_\infty$*(mV)*	
	Nominal	*Worst Case*	*Nominal*	*Worst Case*	*Nominal*	*Worst Case*
Nominal $\mathcal{H}_2$ *with LPTV plant*	*8.95*	-	13.7	-	*72.4*	-
Probabilistic Controller with LPTV plant	*9.48*	*9.87*	12.8	*12.9*	*67.9*	*69.2*

TABLE II

COMPARISON OF THE NOMINAL AND WORST CASE PERFORMANCE SPECIFICATIONS

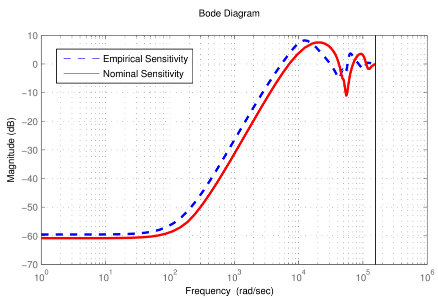

Fig. 7. Closed loop sensitivity function at the output of the controller for LPTV and nominal plant

output sensitivity transfer function, which shows the ability of the system in rejecting different output disturbances, is of vital importance. Although the output signals are not stationary when the plant is LPTV and Controller is LTI their second-order statistics will be periodic, with period N. However for LPTV systems, frequency response methods do not apply. Despite this, we quantified the performance of the closed-loop system by using what we call an "empirical Bode magnitude plot" which uses a swept-sine approach to find an approximate frequency response. The assumption here is that inputting a sine wave into the system produces a negligible response at other frequencies. Note that for an LTI system, the empirical Bode magnitude plot and Bode magnitude plot are equal. Fig. 7 compares the empirical Bode magnitude plot of the sensitivity function of this system, measured at the output of the controller and the nominal controller for the LTI plant. In addition a good way to capture the performance of a controller is to compute the RMS and maximum 3σ values of the relevant signals over a revolution of the HDD. Hence, the Table. II compares the closed loop performance of the system for the nominal and probabilistic controller. A modified version of the standard disturbance [15], which includes repeatable as well as non-repeatable runouts (RRO and NRRO), is used in order to further evaluate the track-following performance of the designed controller. As expected the nominal controller has the best performance for the LTI system however it is not stabilizing when we have 16% sampling time variation. However the probabilistic is stabilizing and the performance of probabilistic controller is 11% degraded with time variation.

V. CONCLUSION

In this paper, we considered a probabilistic approach for robust controller design of HDDs with irregular sampling but regular control updates. We modeled this sampling and actuation behavior by applying a novel discretization method to a continuous-time model of the HDD. Once we had an appropriate discrete-time model of the HDD, we used a probabilistic scenario approach to design a robust controller. We evaluated the performance using a model of the system that included a detailed model of the stochastic disturbances acting on the system. The performance of the controller we designed is only 11% smaller than the limits of performance, while it achieves a high level of robustness in term of stability margins.

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