Mechatronics 20 (2010) 12–19

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

Guaranteed cost control for linear periodically time-varying systems with structured uncertainty and a generalized H2 objective

Richard Conway *, Roberto Horowitz

Computer Mechanics Laboratory, Department of Mechanical Engineering, University of California, Berkeley, CA 94720-1740, United States

ARTICLE INFO

Keywords:
Robust multivariable control
Guaranteed cost control
Dual-stage servos

ABSTRACT

This paper considers the problem of finding a controller for a discrete time linear periodically time-varying system with structured parametric uncertainty which achieves robust ℓ_2 semi-norm performance (the equivalent of robust $\mathscr{H}_2$ norm performance for time-varying systems). This problem is shown to be a generalization of a guaranteed cost control problem when the structure of the uncertainty is neglected. To reduce the conservatism when the uncertainty structure is considered, static uncertainty scalings such as those used in the D-K iteration heuristic for μ-synthesis are introduced. Although the problem of simultaneously optimizing the controller and uncertainty scalings in non-convex, it is shown that it lends itself to a solution methodology conceptually similar to D-K iteration in which each step is a convex optimization. To demonstrate the effectiveness of the developed methodology, the control of a set of hard disk drives with multirate sampling characteristics and uncertain parameters is considered and a controller is designed which minimizes the worst-case ℓ_2 semi-norm performance of the system. It is then shown that the resulting robust controller achieves worst-case ℓ_2 semi-norm performance which is comparable to the best achievable ℓ_2 semi-norm performance for the nominal system.

1. Introduction

$\mathscr{H}_2$ optimal control and $\mathscr{H}_\infty$ optimal control have long been the cornerstones of optimal controller design techniques for discrete time linear time invariant (LTI) systems. The two reasons that these design techniques are so important are that globally optimal controllers can easily be found and the cost functions have intuitive interpretations. In the case of $\mathscr{H}_2$ optimal control, under the assumption that the closed loop system is driven by white Gaussian noise with zero mean and unit covariance, the cost function can be interpreted as the sum of variances of several closed loop signals. This makes it a particularly useful control design technique for LTI systems when the model of the system and its disturbances are precisely known.

A more general class of systems are discrete time linear periodically time-varying (LPTV) systems which admit a state space realization with periodically time-varying entries. With a slight abuse of terminology, we will refer to discrete time LPTV systems which admit a state space realization with periodically time-varying entries simply as LPTV systems. These types of systems arise, for example, when multirate sampling and actuation characteristics are incorporated into an LTI model [1] or if a model is linearized about a periodic trajectory. For LPTV systems, the appropriate generalizations of the $\mathscr{H}_2$ and $\mathscr{H}_\infty$ norms are respectively the ℓ_2 semi-norm (formally defined in the next section) and the ℓ_2 induced norm. The related control design problems can be solved using techniques which are a minor generalization of the techniques for LTI systems (see, for example, [2–4]). Since LTI systems can always be interpreted as LPTV systems with period 1, we will focus our attention on the more general LPTV control problem. In this case, we would like to optimize the ℓ_2 semi-norm performance of the closed loop system.

It is often the case that, although the structure of the plant is known, the model parameters are not. In this case, it is customary to model the uncertainty in the system parameters by bounding their values. The resulting model is then expressed as a linear fractional transformation (LFT) of a known state space system and an unknown norm-bounded diagonal matrix which represents the parametric uncertainty. With this in place, a reasonable control design goal is to find a controller which achieves an adequate level of ℓ_2 semi-norm performance for any system that falls within the modeled uncertainty set.

If the closed loop system is affine in the uncertain parameters, the problem can be formulated as an optimization problem subject to several bilinear matrix inequalities which can be solved using any of the available methodologies [5]. However, since optimization problems subject to bilinear matrix inequalities are non-convex, none of these methodologies guarantee convergence to a

* Corresponding author.
E-mail addresses: rconway345@berkeley.edu (R. Conway), horowitz@me.berkeley.edu (R. Horowitz).

0957-4158/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechatronics.2009.07.003

global optimum. Moreover, since the number of matrix inequalities grows exponentially with the number of uncertain parameters, this approach is not applicable to systems with more than a few uncertain parameters.

An alternative approach to robust performance control design over parametric uncertainty is guaranteed cost control [6]. This methodology is a multiobjective control design methodology whose objectives involve worst-case quadratic costs (in the time domain) over a modeled set of unstructured parametric uncertainty. It has been shown that both the state feedback [6] and the output feedback [7] control problems can be solved via convex optimization involving linear matrix inequalities (LMIs). However, these results are typically conservative because they neglect the structure of the uncertainty. This motivates the usage of uncertainty scalings, as are used in the D-K iteration heuristic for μ-synthesis.

This paper uses the techniques of guaranteed cost control and uncertainty scaling to derive a relevant set of conditions for robust ℓ_2 semi-norm performance of an LPTV system with structured uncertainty. Although the use of dynamic uncertainty scalings is less conservative, we will restrict the uncertainty scalings in this paper to be static to avoid inflation of the controller order during the control design process. It is then shown that although the corresponding controller design problem is non-convex, the problem becomes convex when the values of either of two sets of variables is fixed. Based on this, a control design methodology is presented which is conceptually similar to D-K iteration; the methodology alternates between finding optimal robust ℓ_2 semi-norm performance controllers and optimizing the static uncertainty scalings.

To demonstrate the effectiveness of the developed methodology, we then consider track-following control of hard disk drives (HDDs). For several decades, the areal storage density of HDDs has been doubling roughly every 18 months, as predicted by Kryder's law. As the storage density is pushed higher, the concentric tracks on the disk which contain data must be pushed closer together, which requires more accurate control of the read/write head.

The current goal of the magnetic recording industry is to achieve an areal storage density of 1 terabit/in^2. It is expected that the track width required to achieve this data density is 46 *nm*. This means that the 3σ value of the closed loop position error signal (PES) should be less than 4.6 *nm* to achieve this specification. A typical HDD uses measurements of the PES to control the position of the read/write head via a voice coil motor. To help achieve the desired level of performance, the use of a secondary actuator has been proposed to give increased precision in read/write head positioning. In this paper, we use a microactuator (MA) which directly actuates the head/slider assembly with respect to the suspension tip and generates measurements of the head MA displacement [8]. We will refer to these measurements as the relative position error signal (RPES). To further improve the closed loop performance of the HDD, we would like to increase the sampling and actuation rate of the controller. However, since the sampling rate of the PES is fixed by the rotational speed of the disk and the number of servo sectors written on the disk, we are only free to increase the actuation rate and the sample rate of the RPES while maintaining a fixed PES sampling rate. Since this leads to a control problem with multirate sampling and actuation, we must control an LPTV plant.

However, since there tend to be large variations in HDD dynamics due to variations in manufacturing and assembly, it is not enough to achieve adequate performance for a single plant; the controller must guarantee the desired level of performance for a large set of HDDs. Thus, since we are interested in the variance of the PES, we would like to find a controller which gives robust ℓ_2 semi-norm performance over a set of HDDs. In this paper, we use our proposed control design methodology to design a HDD controller. For comparison, we also design an optimal ℓ_2 semi-norm controller (i.e. a LQG controller) for the nominal system which has no guaranteed robustness properties. We then show that the robust controller achieves performance which is comparable to that achieved by the optimal LQG controller.

2. Guaranteed cost control

2.1. Preliminaries

Throughout this paper, we will be considering discrete time LPTV systems which admit a periodic time-varying state space realization. We will denote the state space realization of an LPTV system, H^d, by

$$H^d \sim \left(\begin{array}{c|c} A_k^d & B_k^d \\ \hline C_k^d & D_k^d \end{array}\right)$$

where the subscript refers to the time index and it is assumed that all state space matrices are periodic with period N, e.g. $A_{k+N}^d = A_k^d$.

In the time domain, if z^{LTI} is the output of an LTI system H^{LTI}, the squared $\mathscr{H}_2$ norm of H^{LTI} has the interpretation of being the sum of the variances of the elements of z^{LTI} when H^{LTI} is driven by zero-mean white Gaussian noise with unit covariance. However, for an LPTV system H^d with output z^d and the same restriction on the its inputs, the second-order statistics of z^d vary periodically with time. Thus, what we are interested in is the average value (over time) of the sum of the variances of the elements of z^d. This corresponds to the definition of the ℓ_2 semi-norm given by

$$\|H^d\|_2^2 := \limsup_{l\to\infty} \frac{1}{2l+1} \sum_{k=-l}^{l} \mathrm{tr}\mathscr{E}[z_k^d (z_k^d)^T].$$

Thus, we are interested in finding an upper bound on the ℓ_2 semi-norm of an LPTV system. The following lemma gives a sharp upper bound on the ℓ_2 semi-norm of an LPTV system [3].

Lemma 1. $\|H^d\|_2^2 < \gamma \iff \exists P_k, W_k$ *such that*

$$\gamma > \frac{1}{N}\sum_{k=1}^{N} \mathrm{tr} W_k, \quad \begin{bmatrix} W_k & C_k^d P_k & D_k^d \\ \cdot & P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0, \quad \begin{bmatrix} P_{k+1} & A_k^d P_k & B_k^d \\ \cdot & P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0$$

for $k = 1, \ldots, N$ *where bullets represent elements which follow from symmetry and* $P_{N+1} = P_1$.

It should also be noted that, for a feasible iterate, the value of W_k represents an upper bound on the LPTV system's output covariance at the associated time steps. With some manipulation, these matrix inequalities can be shown to be equivalent to those obtained using a lifting procedure such as the one used in [2]. The most important difference between the periodic methodology and the lifting methodology is that the matrices in the lifting approach are large and sparse whereas the ones here are dense and have low dimension, i.e. they exploit the relevant sparsity structure. Thus, working with periodic systems will result in more efficient optimization schemes.

Although this paper does not explicitly consider multiobjective control design, one of the goals is to present a methodology which will trivially extend to that case. Thus, we would like to minimize the conservatism of our analysis when performing multiobjective control design. For LTI systems, it was shown that the use of an extended norm characterization using an instrumental variable reduces the conservatism introduced in multiobjective controller design [9]. Thus, in this paper we will be using the following

lemma which can be deduced from the previous one using the methodology in [9].

Lemma 2. $\|H^d\|_2^2 < \gamma \iff \exists P_k, G_k, W_k$ *such that*

$$\gamma > \frac{1}{N}\sum_{k=1}^{N} \mathrm{tr} W_k, \quad \begin{bmatrix} W_k & C_k^d G_k & D_k^d \\ \cdot & G_k + G_k^T - P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0,$$

$$\begin{bmatrix} P_{k+1} & A_k^d G_k & B_k^d \\ \cdot & G_k + G_k^T - P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0$$

for $k = 1, \ldots, N$ *where* $P_{N+1} = P_1$.

2.2. Analysis LMIs

In this section, we develop the necessary theory to derive an upper bound on the worst-case ℓ_2 semi-norm over all modeled uncertainty. First we define the LPTV system

$$H^\Delta \sim \left(\begin{array}{c|c} A_k^\Delta & B_k^\Delta \\ \hline C_k^\Delta & D_k^\Delta \end{array}\right) := \left(\begin{array}{c|c} \overline{A}_k + \Phi_k^{AB}\overline{E}_k^{11} & \overline{B}_k + \Phi_k^{AB}\overline{E}_k^{12} \\ \hline \overline{C}_k + \Phi_k^{CD}\overline{E}_k^{21} & \overline{D}_k + \Phi_k^{CD}\overline{E}_k^{22} \end{array}\right) \tag{1}$$
$$\Phi_k^{AB} := \overline{E}_k^{13}\Delta_k^{AB}\left(I - \overline{E}_k^{14}\Delta_k^{AB}\right)^{-1}, \quad \Phi_k^{CD} := \overline{E}_k^{23}\Delta_k^{CD}\left(I - \overline{E}_k^{24}\Delta_k^{CD}\right)^{-1}$$

where $\overline{A}_k, \overline{B}_k, \overline{C}_k, \overline{D}_k, \overline{E}_k^{ij}$ are known matrices with period N, and the matrices Δ_k^{AB} and Δ_k^{CD} are unknown real matrices (not necessarily periodic) with the form

$$\Delta_k^{AB} \in \underline{\Delta}^{AB} := \{\mathrm{diag}(\alpha^1 I_{m(1)}, \ldots, \alpha^q I_{m(q)}) : |\alpha^i| \leqslant 1\}$$
$$\Delta_k^{CD} \in \underline{\Delta}^{CD} := \{\mathrm{diag}(\beta^1 I_{n(1)}, \ldots, \beta^r I_{n(r)}) : |\beta^i| \leqslant 1\}.$$

This form could be arrived at, for example, by expressing $[A_k^\Delta \; B_k^\Delta]$ as an LFT, expressing $[C_k^\Delta \; D_k^\Delta]$ as an LFT, and then bringing the parametric uncertainty in each LFT inside of its respective loop.

We now define the sets of matrices

$$\underline{S}^{AB} := \left\{\mathrm{diag}(D^1, \ldots D^q) : \begin{array}{l} D^i \in \mathbb{R}^{m(i)\times m(i)} \\ \det D^i \neq 0 \end{array}\right\}, \; \underline{S}^{CD} := \left\{\mathrm{diag}(D^1, \ldots D^r) : \begin{array}{l} D^i \in \mathbb{R}^{n(i)\times n(i)} \\ \det D^i \neq 0 \end{array}\right\}$$

and note that $\Delta_k^{AB} = S^{AB}\Delta_k^{AB}(S^{AB})^{-1}, \; \forall(\Delta_k^{AB}, S^{AB}) \in \underline{\Delta}^{AB} \times \underline{S}^{AB}$, i.e. any matrix in $\underline{S}^{AB}$ commutes with its associated uncertainty, Δ_k^{AB}. Similarly, $\Delta_k^{CD} = S^{CD}\Delta_k^{CD}(S^{CD})^{-1}, \; \forall(\Delta_k^{CD}, S^{CD}) \in \underline{\Delta}^{CD} \times \underline{S}^{CD}$.

For each $S^{AB} \in \underline{S}^{AB}$ and $S^{CD} \in \underline{S}^{CD}$, we now define the sets of unstructured uncertainty

$$\underline{\Delta}^{AB,u}(S^{AB}) := \{\Delta : \|(S^{AB})^{-1}\Delta S^{AB}\| \leqslant 1\},$$
$$\underline{\Delta}^{CD,u}(S^{CD}) := \{\Delta : \|(S^{CD})^{-1}\Delta S^{CD}\| \leqslant 1\}$$

where $\|\Delta\|$ denotes the maximum singular value of Δ. Note that

$$\underline{\Delta}^{AB} \subset \underline{\Delta}^{AB,u}(S^{AB}), \quad \underline{\Delta}^{CD} \subset \underline{\Delta}^{CD,u}(S^{CD}), \qquad \forall S^{AB} \in \underline{S}^{AB}, S^{CD} \in \underline{S}^{CD}. \tag{2}$$

With this notation in place, we can now state and prove two lemmas which we will need to find an upper bound on the guaranteed ℓ_2 semi-norm performance of an uncertain LPTV system.

Lemma 3. *If* $\overline{E}_k^{23} \neq 0$ *and* $S_k \in \underline{S}^{CD}$, *then the following conditions are equivalent:*

1. $\exists P_k, G_k, W_k$ *such that*

$$\begin{bmatrix} W_k & C_k^\Delta G_k & D_k^\Delta \\ \cdot & G_k + G_k^T - P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0, \; \forall \Delta_k^{CD} \in \underline{\Delta}^{CD,u}(S_k) \tag{3}$$

2. $\exists P_k, G_k, W_k, \tau$ *such that*

$$\begin{bmatrix} \tau S_k S_k^T & 0 & \overline{E}_k^{21} G_k & \overline{E}_k^{22} & \tau\overline{E}_k^{24} S_k S_k^T \\ \cdot & W_k & \overline{C}_k G_k & \overline{D}_k & \tau\overline{E}_k^{23} S_k S_k^T \\ \cdot & \cdot & G_k + G_k^T - P_k & 0 & 0 \\ \cdot & \cdot & \cdot & I & 0 \\ \cdot & \cdot & \cdot & \cdot & \tau S_k S_k^T \end{bmatrix} \succ 0 \tag{4}$$

Proof. First we define for convenience

$$\Psi := G_k(G_k + G_k^T - P_k)^{-1}G_k^T,$$
$$L := \begin{bmatrix} 0 & 0 \\ 0 & W_k \end{bmatrix} - \begin{bmatrix} \overline{E}_k^{21} & \overline{E}_k^{22} \\ \overline{C}_k & \overline{D}_k \end{bmatrix}\begin{bmatrix} \Psi & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \overline{E}_k^{21} & \overline{E}_k^{22} \\ \overline{C}_k & \overline{D}_k \end{bmatrix}^T.$$

Via Schur complements, it is easily verified that (3) holds if and only if $\Psi \succ 0$ and

$$\begin{bmatrix} \Phi_k^{CD} & I \end{bmatrix} L \begin{bmatrix} \Phi_k^{CD} & I \end{bmatrix}^T \succ 0, \quad \forall \Delta_k^{CD} \in \underline{\Delta}^{CD,u}(S_k).$$

Defining $\xi := (\Phi_k^{CD})^T x$, the previous condition holds if and only if $\Psi \succ 0$ and

$$\begin{bmatrix} \xi \\ x \end{bmatrix}^T L \begin{bmatrix} \xi \\ x \end{bmatrix} > 0, \quad \forall x \neq 0, \quad \Delta_k^{CD} \in \underline{\Delta}^{CD,u}(S_k). \tag{5}$$

Now note that

$$\xi = [I - (\Delta_k^{CD})^T(\overline{E}_k^{24})^T]^{-1}(\Delta_k^{CD})^T(\overline{E}_k^{23})^T x \Rightarrow \xi = (\Delta_k^{CD})^T \begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix}^T \begin{bmatrix} \xi \\ x \end{bmatrix}$$
$$\Rightarrow S_k^T\xi = (S_k^{-1}\Delta_k^{CD}S_k)^T S_k^T \begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix}^T \begin{bmatrix} \xi \\ x \end{bmatrix}.$$

Thus, $\Delta_k^{CD} \in \underline{\Delta}^{CD,u}(S_k)$ if and only if

$$\xi^T S_k S_k^T \xi \leqslant \begin{bmatrix} \xi \\ x \end{bmatrix}^T \begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix} S_k S_k^T \begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix}^T \begin{bmatrix} \xi \\ x \end{bmatrix}.$$

Since $\overline{E}_k^{23} \neq 0$, $\exists \xi, x$ such that the above inequality is strict. Thus, we can use the S-procedure (see [10]) to say that (5) holds if and only if $\exists \tau > 0$ such that

$$\begin{bmatrix} \xi \\ x \end{bmatrix}^T L \begin{bmatrix} \xi \\ x \end{bmatrix} > \tau \begin{bmatrix} \xi \\ x \end{bmatrix}^T \left(\begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix} S_k S_k^T \begin{bmatrix} \overline{E}_k^{24} \\ \overline{E}_k^{23} \end{bmatrix}^T - \begin{bmatrix} S_k S_k^T & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \xi \\ x \end{bmatrix},$$
$$\forall \xi, x \neq 0.$$

It is straightforward to show using Schur complements that this condition along with the condition $\Psi \succ 0$ is equivalent to (4), which concludes the proof. □

Lemma 4. *If* $\overline{E}_k^{13} \neq 0$ *and* $S_k \in \underline{S}^{AB}$, *then the following conditions are equivalent:*

1. $\exists P_k, G_k$ *such that*

$$\begin{bmatrix} P_{k+1} & A_k^\Delta G_k & B_k^\Delta \\ \cdot & G_k + G_k^T - P_k & 0 \\ \cdot & \cdot & I \end{bmatrix} \succ 0, \quad \forall \Delta_k^{AB} \in \underline{\Delta}^{AB,u}(S_k) \tag{6}$$

2. $\exists P_k, G_k, \tau$ *such that*

$$\begin{bmatrix} \tau S_k S_k^T & 0 & \overline{E}_k^{11} G_k & \overline{E}_k^{12} & \tau\overline{E}_k^{14} S_k S_k^T \\ \cdot & P_{k+1} & \overline{A}_k G_k & \overline{B}_k & \tau\overline{E}_k^{13} S_k S_k^T \\ \cdot & \cdot & G_k + G_k^T - P_k & 0 & 0 \\ \cdot & \cdot & \cdot & I & 0 \\ \cdot & \cdot & \cdot & \cdot & \tau S_k S_k^T \end{bmatrix} \succ 0$$

The proof of this lemma is omitted because it is nearly identical to the proof of the previous lemma.

We now discuss the relevance of the technical condition in Lemma 3 that $\overline{E}_k^{23}$ must be nonzero. Suppose that $\overline{E}_k^{23}=0$. Then $\Phi_k^{CD}=0$, which in turn means that $C_k^{\Delta}=\overline{C}_k$ and $D_k^{\Delta}=\overline{D}_k$. In other words, it is trivial to directly check whether or not (3) holds because C_k^{Δ} and D_k^{Δ} are known. Thus, even though the lemma does not apply, this case can be easily accounted for without introducing conservatism. Similarly, when $\overline{E}_k^{13}=0$, (6) can be checked directly because A_k^{Δ} and B_k^{Δ} are known.

With these two lemmas in place, we can now state and prove the main result of this section, which gives a convex upper bound on the guaranteed ℓ_2 semi-norm of a given system.

Theorem 1. *Assume that a system, H^{Δ}, has the realization (1) and the matrices L and R are given. Then $\|LH^{\Delta}R\|_2^2<\gamma\ \forall(\Delta_k^{AB},\Delta_k^{CD})\in\underline{\Delta}^{AB}\times\underline{\Delta}^{CD}$ if $\exists F_k^{AB}\in\underline{S}^{AB},F_k^{CD}\in\underline{S}^{CD},P_k,G_k,W_k$ such that*

$$\gamma>\frac{1}{N}\sum_{k=1}^{N}\mathrm{tr}W_k \tag{7a}$$

$$\begin{bmatrix} F_k^{CD} & 0 & \overline{E}_k^{21}G_k & \overline{E}_k^{22}R & \overline{E}_k^{24}F_k^{CD} \\ \cdot & W_k & L\overline{C}_kG_k & L\overline{D}_kR & L\overline{E}_k^{23}F_k^{CD} \\ \cdot & \cdot & G_k+G_k^T-P_k & 0 & 0 \\ \cdot & \cdot & \cdot & I & 0 \\ \cdot & \cdot & \cdot & \cdot & F_k^{CD} \end{bmatrix}\succ 0 \tag{7b}$$

$$\begin{bmatrix} F_k^{AB} & 0 & \overline{E}_k^{11}G_k & \overline{E}_k^{12}R & \overline{E}_k^{14}F_k^{AB} \\ \cdot & P_{k+1} & \overline{A}_kG_k & \overline{B}_kR & \overline{E}_k^{13}F_k^{AB} \\ \cdot & \cdot & G_k+G_k^T-P_k & 0 & 0 \\ \cdot & \cdot & \cdot & I & 0 \\ \cdot & \cdot & \cdot & \cdot & F_k^{AB} \end{bmatrix}\succ 0 \tag{7c}$$

for $k=1,\ldots,N$ where $P_{N+1}=P_1$.

Proof. First choose $\tau=1$, perform the Cholesky factorizations $F_k^{AB}=R_k^{AB}(R_k^{AB})^T$, $F_k^{CD}=R_k^{CD}(R_k^{CD})^T$ and note that $R_k^{AB}\in\underline{S}^{AB}$ and $R_k^{CD}\in\underline{S}^{CD}$. We now consider two cases. If $L\overline{E}_k^{23}\neq 0$, we use Lemma 3 to conclude that

$$\begin{bmatrix} W_k & LC_k^{\Delta}G_k & LD_k^{\Delta}R \\ \cdot & G_k+G_k^T-P_k & 0 \\ \cdot & \cdot & I \end{bmatrix}\succ 0,\quad \forall\Delta_k^{CD}\in\underline{\Delta}^{CD,u}(R_k^{CD}). \tag{8}$$

If $L\overline{E}_k^{23}=0$, we note that since $LC_k^{\Delta}=L\overline{C}_k$ and $LD_k^{\Delta}=L\overline{D}_k$, the $2^{nd},3^{rd}$, and 4th rows and columns of (7b) are equivalent to (8). Thus, in either case, we conclude that (8) holds. Similarly,

$$\begin{bmatrix} P_{k+1} & A_k^{\Delta}G_k & B_k^{\Delta}R \\ \cdot & G_k+G_k^T-P_k & 0 \\ \cdot & \cdot & I \end{bmatrix}\succ 0,\quad \forall\Delta_k^{AB}\in\underline{\Delta}^{AB,u}(R_k^{AB}). \tag{9}$$

Fixing $\Delta_k^{AB}\in\underline{\Delta}^{AB}$, $\Delta_k^{CD}\in\underline{\Delta}^{CD}$ and using Lemma 2 with (7a), (8), (9) and (2) concludes the proof. □

Before using Theorem 1, it should always be checked whether or not $\overline{E}_k^{13}$ and/or $L\overline{E}_k^{23}$ are nonzero. There are two reasons for this. For instance, suppose that $\overline{E}_k^{13}=0$. In this case, using condition (7c) would introduce unnecessary conservatism into the analysis, i.e. our analysis would be less conservative if we instead directly checked (6). Second, from a computational standpoint, it makes more sense in this case to check (6) because that LMI has smaller dimensions and contains fewer variables than (7c). Similar considerations apply when $L\overline{E}_k^{23}=0$. Thus, when $L\overline{E}_k^{23}=0$, (7b) in Theorem 1 should be replaced by its $2^{nd},3^{rd}$, and 4th rows and columns. Similarly when $\overline{E}_k^{13}=0$, (7c) in Theorem 1 should be replaced by its $2^{nd},3^{rd}$, and 4th rows and columns.

2.3. Output feedback controller design

In this section, we apply the generalization of the Lyapunov shaping paradigm presented in [9] to controller design using the LMIs in Theorem 1. First, we let the uncertain LPTV plant, H^u, and the controller respectively have the form

$$\begin{bmatrix} x_{k+1} \\ z_k^{\infty} \\ z_k \\ y_k \end{bmatrix}=\left[\begin{array}{c|ccc} A_k & B_k^1 & B_k^2 & B_k^3 \\ \hline C_k^1 & D_k^{11} & D_k^{12} & D_k^{13} \\ C_k^2 & D_k^{21} & D_k^{22} & D_k^{23} \\ C_k^3 & D_k^{31} & D_k^{32} & 0 \end{array}\right]\begin{bmatrix} x_k \\ w_k^{\infty} \\ w_k \\ u_k \end{bmatrix},\quad \begin{bmatrix} x_{k+1}^K \\ u_k \end{bmatrix}=\left[\begin{array}{c|c} A_k^K & B_k^K \\ \hline C_k^K & D_k^K \end{array}\right]\begin{bmatrix} x_k^K \\ y_k \end{bmatrix} \tag{10}$$

where the plant uncertainty is represented as

$$w_k^{\infty}=\Delta_k z_k^{\infty},$$
$$\Delta_k\in\underline{\Delta}:=\{\mathrm{diag}(\alpha^1 I_{m(1)},\ldots,\alpha^{q'}I_{m(q')}):\alpha^i\in[-1,1]\}. \tag{11}$$

It should be noted that since the controller is chosen to be a state space system with periodically time-varying entries. When the controller is brought inside the loop, the plant has the state space form

$$\begin{bmatrix} \bar{x}_{k+1} \\ z_k^{\infty} \\ z_k \end{bmatrix}=\left[\begin{array}{c|cc} \overline{A}_k & \overline{E}_k^{13} & \overline{B}_k \\ \hline \overline{E}_k^{x1} & \overline{E}_k^{x4} & \overline{E}_k^{x2} \\ \overline{C}_k & \overline{E}_k^{23} & \overline{D}_k \end{array}\right]\begin{bmatrix} \bar{x}_k \\ w_k^{\infty} \\ w_k \end{bmatrix},\quad w_k^{\infty}=\Delta_k z_k^{\infty},\quad \Delta_k\in\underline{\Delta}$$

where the new state is given by $\bar{x}_k^T=[x_k^T\ (x_k^K)^T]$ and the state space matrices are given by

$$\overline{A}_k:=\begin{bmatrix} A_k+B_k^3D_k^KC_k^3 & B_k^3C_k^K \\ B_k^KC_k^3 & A_k^K \end{bmatrix},$$
$$\begin{bmatrix}\overline{E}_k^{13} & \overline{B}_k\end{bmatrix}:=\begin{bmatrix} B_k^1+B_k^3D_k^KD_k^{31} & B_k^2+B_k^3D_k^KD_k^{32} \\ B_k^KD_k^{31} & B_k^KD_k^{32}\end{bmatrix}$$
$$\begin{bmatrix}\overline{E}_k^{x1} \\ \overline{C}_k\end{bmatrix}:=\begin{bmatrix} C_k^1+D_k^{13}D_k^KC_k^3 & D_k^{13}C_k^K \\ C_k^2+D_k^{23}D_k^KC_k^3 & D_k^{23}C_k^K\end{bmatrix},$$
$$\begin{bmatrix}\overline{E}_k^{x4} & \overline{E}_k^{x2} \\ \overline{E}_k^{23} & \overline{D}_k\end{bmatrix}:=\begin{bmatrix} D_k^{11}+D_k^{13}D_k^KD_k^{31} & D_k^{12}+D_k^{13}D_k^KD_k^{32} \\ D_k^{21}+D_k^{23}D_k^KD_k^{31} & D_k^{22}+D_k^{23}D_k^KD_k^{32}\end{bmatrix}.$$

When the time-varying gain Δ_k is also brought inside the loop, it results in the realization in (1) with

$$\overline{E}_k^{11}=\overline{E}_k^{21}=\overline{E}_k^{x1},\quad \overline{E}_k^{12}=\overline{E}_k^{22}=\overline{E}_k^{x2},\quad \overline{E}_k^{14}=\overline{E}_k^{24}=\overline{E}_k^{x4}.$$

At this point, we apply the generalization of the Lyapunov shaping paradigm presented in [9]. This results in the following expressions for the terms in (7b) and (7c):

$$\overline{A}_kG_k=\begin{bmatrix} A_kX_k+B_k^3\widehat{C}_k & A_k+B_k^3\widehat{D}_kC_k^3 \\ \widehat{A}_k & Y_{k+1}A_k+\widehat{B}_kC_k^3\end{bmatrix},\quad \overline{B}_k=\begin{bmatrix} B_k^2+B_k^3\widehat{D}_kD_k^{32} \\ Y_{k+1}B_k^2+\widehat{B}_kD_k^{32}\end{bmatrix},$$
$$\overline{C}_kG_k=\begin{bmatrix} C_k^2X_k+D_k^{23}\widehat{C}_k & C_k^2+D_k^{23}\widehat{D}_kC_k^3\end{bmatrix},\quad \overline{D}_k=\begin{bmatrix} D_k^{22}+D_k^{23}\widehat{D}_kD_k^{32}\end{bmatrix},$$
$$\overline{E}_k^{13}=\begin{bmatrix} B_k^1+B_k^3\widehat{D}_kD_k^{31} \\ Y_{k+1}B_k^1+\widehat{B}_kD_k^{31}\end{bmatrix},\quad \overline{E}_k^{23}=\begin{bmatrix} D_k^{21}+D_k^{23}\widehat{D}_kD_k^{31}\end{bmatrix},$$
$$\overline{E}_k^{11}=\overline{E}_k^{21}=\begin{bmatrix} C_k^1X_k+D_k^{13}\widehat{C}_k & C_k^1+D_k^{13}\widehat{D}_kC_k^3\end{bmatrix},$$
$$\overline{E}_k^{12}=\overline{E}_k^{22}=\begin{bmatrix} D_k^{12}+D_k^{13}\widehat{D}_kD_k^{32}\end{bmatrix},$$
$$\overline{E}_k^{14}=\overline{E}_k^{24}=\begin{bmatrix} D_k^{11}+D_k^{13}\widehat{D}_kD_k^{31}\end{bmatrix},G_k=\begin{bmatrix} X_k & I \\ Z_k & Y_k\end{bmatrix},\quad P_k=\begin{bmatrix} P_k^1 & P_k^2 \\ \cdot & P_k^3\end{bmatrix}. \tag{12}$$

First note that the right-hand sides of all these equalities are affine in $P_k^1,P_k^2,P_k^3,X_k,Y_k,Z_k,\widehat{A}_k,\widehat{B}_k,\widehat{C}_k,\widehat{D}_k$. Also note that, W_k,F_k^{AB}, and F_k^{CD} remain unchanged after applying the generalized Lyapunov shaping

paradigm. Thus, utilizing Theorem 1 with the expressions in (12) gives a set of matrix inequalities in the optimization variables $W_k, P_k^1, P_k^2, P_k^3, X_k, Y_k, Z_k, \widehat{A}_k, \widehat{B}_k, \widehat{C}_k, \widehat{D}_k, F_k^{AB}$, and F_k^{CD}. (P_k^1 and P_k^3 are symmetric and all other variables are full.) For a given set of optimization variable values, the corresponding controller is given by

$$\begin{bmatrix} A_k^K & B_k^K \\ C_k^K & D_k^K \end{bmatrix} = \begin{bmatrix} N_{k+1}^{-1} & -N_{k+1}^{-1} Y_{k+1} B_k^3 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_k - Y_{k+1} A_k X_k & \widehat{B}_k \\ \widehat{C}_k & \widehat{D}_k \end{bmatrix} \times \begin{bmatrix} M_k^{-1} & 0 \\ -C_k^3 X_k M_k^{-1} & I \end{bmatrix} \tag{13}$$

$$N_k M_k = Z_k - Y_k X_k \tag{14}$$

where N_k and M_k can be any matrices satisfying (14). It should be noted that the resulting controller is an LPTV state space system with the same period and number of states as the plant.

Note that if we fix F_k^{AB} and F_k^{CD}, the matrix inequalities in Theorem 1 become affine in the above optimization parameters. This means that minimizing the guaranteed ℓ_2 semi-norm performance of the system using this formulation is convex when these variables are fixed. Alternatively, when $\widehat{B}_k, \widehat{D}_k$, and Y_k are fixed, the matrix inequalities become affine in the above optimization parameters, again resulting in a convex optimization to minimize the guaranteed ℓ_2 semi-norm performance of the system. Based on these two facts, we can construct the following methodology for control design:

Step 1: *Initial Controller Design:* Find a controller of the same order as the plant which achieves robust stability over some unstructured uncertainty set, $\underline{\Delta}(S_k)$. This could be done using D-K iteration or $\mathscr{H}_\infty$ optimal control.

Step 2: *Initial Uncertainty Scalings:* First fix the controller found in the previous step and bring it inside the loop. Then use Theorem 1 to find permissible values of P_k, G_k, W_k, F_k^{AB}, and F_k^{CD}. (Do not use (12) in this step.)

Step 3: *Control Design:* Fix the value of F_k^{AB} and F_k^{CD} found in the previous step and minimize the guaranteed ℓ_2 semi-norm performance using convex optimization applied to Theorem 1 with (12).

Step 4: *Uncertainty Scaling:* Fix the values of $\widehat{B}_k, \hat{D}_k$, and Y_k found in the previous step and minimize the guaranteed ℓ_2 semi-norm performance using convex optimization applied to Theorem 1 with (12).

Step 5: *Check Stop Criterion:* Check a relevant stopping criterion. If it is not met, return to Step 3.

Step 6: *Reconstruct Controller:* Use (13) and (14) to reconstruct the controller.

In this methodology, there are two subtleties that a good implementation should exploit during a preprocessing phase. First, to reduce conservatism, it should be checked for each LMI at each time step whether or not the uncertainty scalings are necessary. For instance, if $LD_k^{21}, D_k^{31} = 0$, then $L\bar{E}_k^{23} = 0$ and the generalized Lyapunov shaping paradigm should be applied to only the $2^{nd}, 3^{rd}$, and 4th rows and columns in (7b) for that time index. Second, the coupling between variables should be checked at each time step. For instance, if $D_k^{31} = 0$, then F_k^{AB} and F_k^{CD} are decoupled from $\widehat{B}_k$ and $\widehat{D}_k$. Thus, in step 4 of the methodology above, it is only necessary to fix Y_k for that particular time index.

This methodology is similar to D-K iteration because in both, the controller design process alternates between finding controllers which achieve optimal performance (for a slightly conservative problem) and optimizing the uncertainty scalings. There is one benefit, however, that this methodology has over D-K iteration. Although it is necessary to fix the controller when optimizing the uncertainty scalings in D-K iteration, only some of the control parameters need to be fixed when optimizing the static uncertainty scalings in this methodology. Moreover, in some cases, none of the control parameters need to be fixed in order to scale the uncertainty.

Although this paper does not consider multiobjective control design, all of the techniques presented here could be trivially extended to allow for control design with multiple guaranteed ℓ_2 semi-norm performance objectives. In particular, we would apply Theorem 1 to each objective, i.e. each choice of (L, R) in Theorem 1. In this case, since our controller reconstruction does not depend on the values on $W_k, F_k^{AB}, F_k^{CD}, P_k^1, P_k^2$, and P_k^3, these variables can be allowed to be constraint-dependent, i.e. each application of Theorem 1 with (12) could have different values for these variables. For more details on convex multiobjective control design, refer to [11,9].

3. Track-following control

In this section, we consider the track following control of a hard disk drive with dual-stage actuation as an application of the control methodology presented in this paper. Fig. 1 shows the structure and block diagram of a hard disk drive with dual-stage actuation where the signal units and descriptions are summarized in Table 1. In the block diagram, H, K, and W_r respectively represent the HDD, the controller, and the coloring filter which generates r from white noise. The closed loop inputs (w^r, w^a, n^{PES}, and n^{RPES}) are independent Gaussian white noises with unit coveriance and zero mean. The closed loop outputs in this block diagram represent the quantities that we would like to keep small in the closed loop system—the head position error ($y^h - r$), the MA displacement ($y^h - y^s$), and the control inputs (u^v and u^m). Although we would like to keep the head position error and the control inputs as small as possible, the MA displacement only needs to be kept smaller than the MA stroke of $1\mu m$.

The discrete time LTI model of H with sampling frequency 50 kHz is given by

$$\begin{bmatrix} y^h(z) \\ y^s(z) \end{bmatrix} := \sum_{i=1}^{4} \begin{bmatrix} 1 & 0 \\ c^{1,i} & c^{2,i} \end{bmatrix} \left(zI - \begin{bmatrix} a^{1,i} & 1 \\ a^{2,i} & 0 \end{bmatrix} \right)^{-1} B^i \begin{bmatrix} w^a(z) \\ u^v(z) \\ u^m(z) \end{bmatrix},$$

$$a^{j,i} := \bar{a}^{j,i} + m^{j,i}\delta^i, \quad \delta^i \in [-1, 1]$$

where the model parameters are summarized in Table 2. It should be noted that the transfer function from u^m to y^s is 0, i.e. the MA control has a negligible effect on the suspension displacement. Also, note that this model is expressed in modal form where the discrete time natural frequency and damping for each mode are uncertain. The model has three normalized uncertainties: δ^2, δ^3, and δ^4. Each one of these uncertainties only affects the natural frequency and damping for its associated mode. The model of W_r is given by

$$W_r(z) = \frac{2.355}{z - 0.9627} + \frac{0.557z + 0.5541}{z^2 - 1.984z + 0.9841}.$$

Frequency responses of H and W_r for 50 random models in the uncertain set are shown in Fig. 2.

With some standard manipulation, this model can be put into the form of (10) and (11) where

$$z_k := \begin{bmatrix} y_k^h - r_k \\ y_k^h - y_k^s \\ u_k^v \\ u_k^m \end{bmatrix}, \quad w_k := \begin{bmatrix} w_k^r \\ w_k^a \\ n_k^{PES} \\ n_k^{RPES} \end{bmatrix},$$

$$y_k := \begin{bmatrix} y_k^h - r_k + n_k^{PES} \\ y_k^h - y_k^s + n_k^{RPES} \end{bmatrix}, \quad u_k := \begin{bmatrix} u_k^v \\ u_k^m \end{bmatrix}.$$

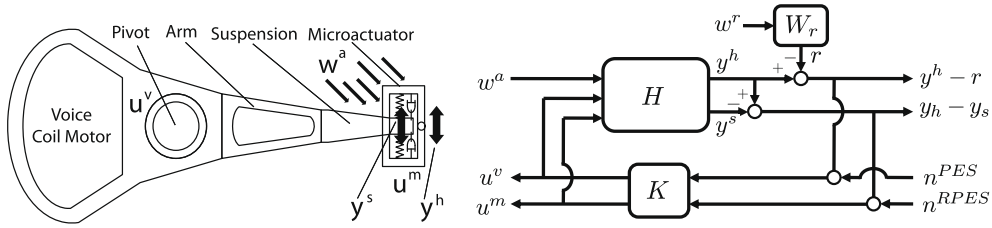

Fig. 1. Schematic and block diagram of the dual-stage hard drive.

Table 1
Description and units of hard drive model signals.

Signal	Description	Units
n^{PES}	PES sensor noise	nm
n^{RPES}	RPES sensor noise	nm
r	Disturbances on the head position [12]	nm
u^v	Voice coil motor control	V
u^m	Microactuator control	V
w^r	White noise which generates r	(normalized)
w^a	Airflow disturbances	(normalized)
y^h	Head displacement relative to the track center	nm
y^s	Suspension tip displacement	nm

The PES measurement noise is relatively large in this model (1 *nm* at 1σ) because we are assuming that we are measuring the PES via a laser doppler velocimeter, as we will do when we try to implement the controller in the future.

In our final controller design, although we will sample the RPES and actuate our system at the rate of 50 kHz, we are bound by the constraint that the PES can only be sampled at the rate of 25 kHz. Thus, as discussed in [13], the signal which should be sent to our controller is $\bar{y}_k = \Omega_k y_k$ where $\Omega_k = I$ at odd time steps and $\Omega_k = \mathrm{diag}(0, 1)$ at even steps. Note that because Ω_k is an LPTV gain with period 2, it results in an LPTV system with period 2 when brought inside the model (i.e. when y_k in (10) is replaced by $\bar{y}_k$).

With this in place, we chose the output and input weights to respectively be $L = \mathrm{blkdiag}([1\ 0], 1, 1)$ and $R = I$ and minimized the guaranteed ℓ_2 semi-norm performance of the closed loop system by using the control design methodology presented in this paper. It should be noted that this choice of L does not penalize the MA displacement, $y^h - y^s$. The optimizations were performed using the `mincx` command in the Robust Control Toolbox for `MATLAB`. The resulting controller was a state space system of order 11 in which the state space entries were periodically time-varying with period 2. Although this paper does not consider the implementation of this controller on an actual hardware setup, it should be noted that this controller would be implemented in a fashion similar to that of an LTI state space controller.

Table 2
Hard drive model parameters.

mode i	$\begin{bmatrix}\bar{a}^{1,i}\\ \bar{a}^{2,i}\end{bmatrix}$	$\begin{bmatrix}m^{1,i}\\ m^{2,i}\end{bmatrix}$	$\begin{bmatrix}c^{1,i}\\ c^{2,i}\end{bmatrix}$	B^i
1	$\begin{bmatrix}1.9924\\ -0.9925\end{bmatrix}$	$\begin{bmatrix}0\\ 0\end{bmatrix}$	$\begin{bmatrix}0.9054\\ -0.0946\end{bmatrix}$	$\begin{bmatrix}0 & 0.4716 & 0\\ 0 & 0.3212 & 0\end{bmatrix}$
2	$\begin{bmatrix}1.1818\\ -0.9726\end{bmatrix}$	$\begin{bmatrix}0.1171\\ -0.0071\end{bmatrix}$	$\begin{bmatrix}0.7883\\ 6.5939\end{bmatrix}$	$\begin{bmatrix}0 & 0.1239 & 0\\ 0 & -0.0655 & 0\end{bmatrix}$
3	$\begin{bmatrix}0.4461\\ -0.9605\end{bmatrix}$	$\begin{bmatrix}0.1995\\ -0.0102\end{bmatrix}$	$\begin{bmatrix}-1.9184\\ 9.8265\end{bmatrix}$	$\begin{bmatrix}-0.0678 & 0.0353 & 0\\ 0.0317 & -0.0165 & 0\end{bmatrix}$
4	$\begin{bmatrix}1.8188\\ -0.8937\end{bmatrix}$	$\begin{bmatrix}0.0455\\ -0.0299\end{bmatrix}$	$\begin{bmatrix}0\\ 0\end{bmatrix}$	$\begin{bmatrix}0.1111 & -0.6329 & 1.5314\\ -0.0209 & -0.2080 & 1.4749\end{bmatrix}$

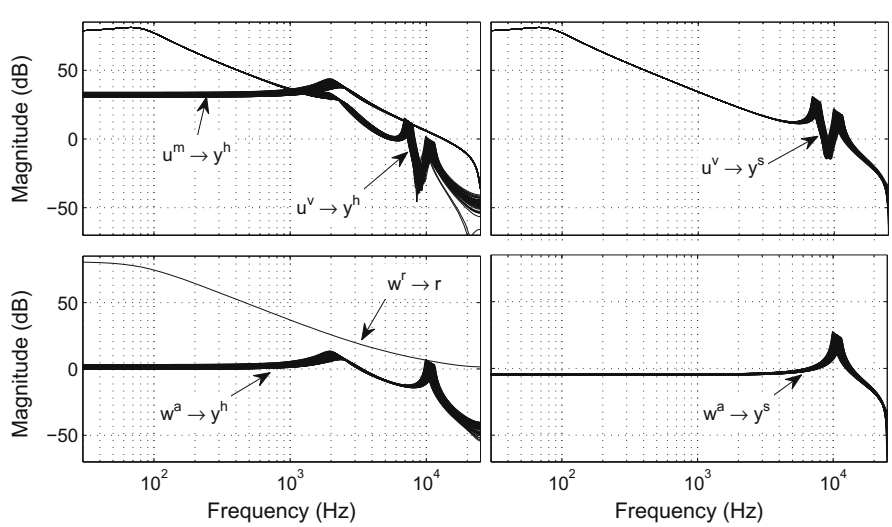

Fig. 2. Relevant hard drive frequency responses.

As previously mentioned, the values of W_k returned by the optimization routine represent an upper bound on the output covariance of the closed loop system at the corresponding time steps. Therefore, the elements of $0.5(W_1 + W_2)$ represent an upper bound on the worst-case (over uncertainty) time averaged covariance of the closed loop values of the system output, Lz_k. From this, estimates of the worst-case ℓ_2 semi-norm performance of these signals were generated. Table 3 shows these values. Note that because $y^h - y^s$ was not penalized in the control design, no estimate of its worst-case value was generated.

Since the optimization and controller reconstruction routines could potentially be adversely affected by numerical errors, we verified these upper bounds by performing a Monte Carlo analysis of the ℓ_2 semi-norm performance of each output signal of our uncertain closed loop system. This analysis consisted of two parts – examining the performance of the nominal system and examining the worst-case performance (for each signal in z_k) over 400 random samples of the closed loop system. The ℓ_2 semi-norm performance of each signal in each closed loop system sample was found by computing the relevant $\mathscr{H}_2$ norm of the lifted LTI system [3]. The results of this analysis are shown in Table 4. Since the worst-case performance over the 400 random system samples is better than the results returned by the optimization routine, we conclude that the optimization results are meaningful. Moreover, since the achieved worst-case MA displacement is two orders of magnitude smaller than the MA stroke, the decision to not penalize $y^h - y^s$ in the control design is justified. Also, by examining Fig. 3, which shows a sample sequence of the head position error and its corresponding FFT magnitude for a random sample of the closed loop system, we see that the multirate sampling and periodicity of the controller is not adversely affecting the performance of the system. However, since the bound on u^v appears to be rather conservative in the Monte Carlo analysis, it remains to be seen whether or not the achieved performance is also conservative.

Table 3
Worst-case root mean square (over time) 1σ signal values for robust ℓ_2 semi-norm design, as reported by solver.

Signal	$y^h - r$ (nm)	$y^h - y^s$ (nm)	u^v (V)	u^m (V)
Value	5.0480	N/A	3.2536	1.8923

Table 4
Monte Carlo analysis of root mean square (over time) 1σ signal values for robust ℓ_2 semi-norm design.

	$y^h - r$ (nm)	$y^h - y^s$ (nm)	u^v (V)	u^m (V)
Nominal	4.8133	9.7314	1.7252	1.8014
Worst-Case	4.8969	10.4680	1.8538	1.8195

Table 5
Monte Carlo analysis of root mean square (over time) 1σ signal values for nominal ℓ_2 semi-norm design.

	$y^h - r$ (nm)	$y^h - y^s$ (nm)	u^v (V)	u^m (V)
Nominal	4.7784	20.0021	0.5803	1.895
Worst-Case	4.9852	27.4165	0.6547	2.0055

For comparison, we also designed an optimal ℓ_2 semi-norm (i.e. LQG) controller for the nominal system using the methodology in [4]. It should be noted that because LQG controllers have no guaranteed stability margins, this is not a recommended technique for controller design for HDDs. Despite this, we used this controller to close the loop in the uncertain model and then performed the same Monte Carlo analysis as before. Note that the nominal performance in this case refers to the optimal performance which is achievable by the nominal system. Also note that, because $y^h - y^s$ is not penalized in this design, it is allowed to have a relatively large value. The results of the Monte Carlo analysis are shown in Table 5. For the uncertain plant considered here, the Monte Carlo analysis indicates that the LQG controller has a reasonable level of robustness. As expected, the worst-case 1σ head position error is larger in the LQG design. And, although the 1σ head position error is smallest in the LQG design applied to the nominal system, the nominal and worst-case 1σ head position error values in the robust design are respectively less than 1% and 2.5% worse than this value. Thus, despite the bound on u^v being rather conservative, we see that the resulting controller achieves a high level of performance, even in the worst case.

4. Conclusion

In this paper, we have proposed a new method for designing controllers which achieve robust ℓ_2 semi-norm performance. The control design methodology proposed, although it is not guaranteed to find a controller which locally minimizes guaranteed ℓ_2 semi-norm performance over the structured uncertainty set (i.e. it globally optimizes the guaranteed ℓ_2 semi-norm performance over some, slightly conservative, unstructured uncertainty set), it is conceptually similar to the D-K iteration heuristic for μ-synthesis which has been successful in many control design applications. However, unlike D-K iteration, the methodology presented here does not require all of the control parameters to be fixed in order to optimize the static uncertainty scalings. This design methodology was used to design a HDD controller with multi-rate sampling and actuation characteristics which achieves

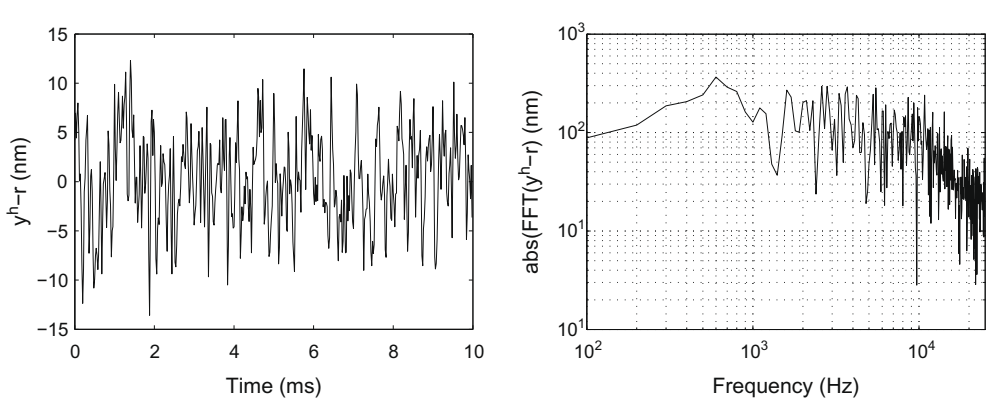

Fig. 3. Sample sequence of the closed loop head position error and its FFT magnitude.

performance comparable to the optimal LQG controller for the nominal system.

Acknowledgement

This work was supported in part by National Science Foundation Grant CMS-0428917, the Information Storage Industry Consortium and the Computer Mechanics Laboratory at UC Berkeley.

References

[1] Crochiere, RE, Rabiner, LR. Interpolation and decimation of digital signals – a tutorial review. In: Proceedings of the IEEE, vol. 69; 1981. p. 300–31.
[2] Dullerud GE, Lall S. A new approach for analysis and synthesis of time-varying systems. IEEE Trans Autom Control 1999;44(8):1486–97.
[3] Conway R. Multi-objective control design for discrete time periodic systems via convex optimization. Tech. rep., University of California at Berkeley; 2008.
[4] Conway R, Horowitz R. A quasi-Newton algorithm for LQG controller design with variance constraints. In: Proceedings of the DSCC; 2008.
[5] Kanev S, Scherer C, Verhaegen M, De Schutter B. Robust output-feedback controller design via local BMI optimization. Automatica 2004;40(7):1115–27.
[6] Petersen IR, McFarlane DC, Rotea MA. Optimal guaranteed cost control of discrete-time uncertain linear systems. International Journal of Robust and Nonlinear Control 1998;8(8):649–57.
[7] Yu L, Gao F. Output feedback guaranteed cost control for uncertain discrete-time systems using linear matrix inequalities. Journal of Optimization Theory and Applications 2002;113(3):621–34.
[8] Oldham K, Huang X, Chahwan A, Horowitz R. Design, fabrication, and control of a high-aspect ratio microactuator for vibration suppression in a hard disk drive. In: Proceedings of the 16th IFAC World Congress; 2005.
[9] de Oliveira MC, Geromel JC, Bernussou J. Extended H_2 and H_∞ norm characterizations and controller parametrizations for discrete-time systems. International Journal of Control 2002;75(9):666–79.
[10] Boyd S, El Ghaoui L, Feron E, Balakrishnan V. Linear matrix inequalities in system and control theory. SIAM Stud Appl Math, vol. 15. Philadelphia: Society for Industrial and Applied Mathematics; 1994.
[11] Scherer C, Gahinet P, Chilali M. Multiobjective output-feedback control via LMI optimization. IEEE Trans Autom Control 1997;42(7):896–911.
[12] Huang X, Nagamune R, Horowitz R. A comparison of multirate robust track-following control synthesis techniques for dual-stage and multi-sensing servo systems in hard disk drives. IEEE Trans Magnet 2006;42(7):1896–904.
[13] Nagamune R, Huang X, Horowitz R. Multirate track-following control with robust stability for a dual-stage multi-sensing servo system in HDDs, In: Proceedings of the joint 44th IEEE conference on decision and control and European control conference; 2005. p. 3886–3891.